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ABSTRACT

This collection of 11 applied algebra curriculum modules can be used independently as supplemental modules for an existing algebra curriculum. They represent diverse curriculum styles that should stimulate the teacher's creativity to adapt them to other algebra concepts. The selected topics have been determined to be those most needed by students in both vocational-technical and academic programs. Topics are as follows: (1) real number properties and operations; (2) problem solving--geometric figures; (3) graphing skills; (4) exponents and roots; (5) estimation skills; (6) word problems; (7) problem solving--rates; (8) linear equations and inequalities; (9) quadratic equations and inequalities; (10) functions; and (11) use of statistics. Modules 1, 2, 8, and 9 consist of these components: objectives; equipment list; handouts/activity or exercise sheets; and informative material for the teacher. Modules 3, 5, and 10 have this format: performance objective, investigations/demonstrations each followed by an activity, evaluation instrument, and list of required materials. Module 4 follows this format: performance objective, background information, demonstrations followed by activities, handouts, workplace/technical problems, posttest, and equipment/materials list. Modules 6 and 7 have these components: performance objective, statement of connection, activity, list of evaluation instruments, and supply list. Module 11 follows this format: introduction, materials list, lesson plan, handouts, list of course objectives, skill check with answer key, and glossary. (YLB)

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ED 395 185

Applied Algebra Curriculum Modules

Texas State Technical College - East Texas Center

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Dear Algebra Instructor:

This collection of **Applied Algebra Curriculum Modules** is a product of Carl D. Perkins Project # 55170025, **Intermediate / College Algebra Applied Methodologies & Accelerated Learning.** Each module can be used independently. If you choose to use the modules in your course of instruction, feedback from you and your students regarding the module(s) would be appreciated and retained for future revisions of the modules.

For this purpose, an attitudinal **PRE-SURVEY** and **POST-SURVEY** form has been inserted after the **Table of Contents** page for your use. Please use the form as a **2-sided master** and duplicate as many forms as you require for your class(es). Your general and specific comments would also be most helpful to future revisions and/or additions. **Please specify which module(s) was(were) used in your instructor comments.**

The completed surveys and/or instructor comments should be returned to:

Mr. Harvey Fox, Program Director
Texas State Technical College - East Texas Center
P.O. Box 1269
Marshall, TX 75671

If you have any questions, please contact Harvey Fox at **903-935-1010.**

Thank you for your assistance in this project.

Applied Algebra Curriculum Modules

TABLE OF CONTENTS

1. Overview
2. Real Number Properties and Operations
3. Problem Solving - Geometric Figures
4. Graphing Skills
5. Exponents and Roots
6. Estimation Skills
7. Word Problems
8. Problem Solving - Rates
9. Linear Equations and Inequalities
10. Quadratic Equations and Inequalities
11. Functions
12. Use of Statistics

Carl D. Perkins Project # 55170025
Intermediate / College Algebra Applied Methodologies & Accelerated Learning

PRE-SURVEY

BACKGROUND INFORMATION:

Check one of the following:

_____ I have **passed** the Math portion of the TASP Test.

_____ I have **not passed** the Math portion of the TASP Test.

_____ I am **exempt** from the TASP test.

FILL IN THE BLANKS:

My current math course is ______________________________.

I took my last high school algebra course ________ years ago.**
**** Note - Write "never" in the blank if you have never taken a high school algebra course.**

PLEASE RESPOND TO THE FOLLOWING STATEMENTS WITH:

1 = strongly disagree,
2 = mildly disagree,
3 = mildly agree,
4 = strongly agree.

STATEMENTS:

_____ **1. I easily catch on to mathematical concepts.**

_____ **2. I learn mathematics concepts with any method of instruction.**

_____ **3. I prefer traditional mathematics instruction involving lecture and practice.**

_____ **4. I see a purpose for learning mathematical concepts.**

_____ **5. I am motivated to learn mathematical concepts.**

PLEASE COMPLETE EACH STATEMENT BELOW.

6. When I have had **difficulties** in math, the main reason for the difficulties has been ______________________________

______________________________.

7. When I have been **successful** in math, the main reason for my success has been ______________________________

______________________________.

END OF PRE-SURVEY

*****PLEASE RETAIN THIS PAGE FOR LATER USE*****

POST-SURVEY

PLEASE RESPOND TO THE FOLLOWING STATEMENTS WITH:

1 = strongly disagree,
2 = mildly disagree,
3 = mildly agree,
4 = strongly agree.

STATEMENTS:

_____ 1. The APPLIED ALGEBRA CURRICULUM MODULE helped me to catch on to algebra concepts better than traditional lecture instruction.

_____ 2. The methods of instruction used in the APPLIED ALGEBRA CURRICULUM MODULE are most effective for me.

_____ 3. I would like to see my math instructors use methods of instruction like those used in the APPLIED ALGEBRA CURRICULUM MODULE for teaching mathematics in my future math courses.

_____ 4. With the APPLIED ALGEBRA CURRICULUM MODULE, I readily see a purpose for algebra concept(s).

_____ 5. The APPLIED ALGEBRA CURRICULUM MODULE motivated me to learn algebra concepts better than traditional lecture instruction.

_____ 6. I am confident that I can successfully use the algebra concept(s) I have just been taught in the APPLIED ALGEBRA CURRICULUM MODULE.

PLEASE COMPLETE EACH STATEMENT BELOW.

7. What I liked **most** about the **APPLIED ALGEBRA CURRICULUM MODULE** is

____.

8. What I liked **least** about the **APPLIED ALGEBRA CURRICULUM MODULE** is

____.

END OF POST-SURVEY
THANK YOU FOR YOUR COOPERATION!

OVERVIEW

Applied Algebra Curriculum Modules

This collection of Applied Algebra Curriculum Modules has been sent to you in the hopes that you will find them to be directly usable as supplemental modules for your existing algebra curricula. They also represent a series of diverse curriculum styles which hopefully will stimulate your creative spirit to adapt them to other algebra concepts. While each module is designed to supplement your existing course of study, they can be used as stand-alone units, although they clearly do not constitute a complete course. The selected topics have been determined to be those most needed by students in both technical and academic programs.

The six authors of these modules represent a variety of educational levels as indicated by the institutions where they work. These levels range from ninth grade algebra through more advanced high school mathematics to community college, technical college, and industrial apprenticeship training. Likewise, the Technical Advisory Committee for the Carl D. Perkins Project #55170025 which funded these modules, also represents secondary level mathematics, community and technical colleges mathematics, four-year university mathematics, and industrial workplace training programs.

The overall focus of the project is to develop and implement applications methodologies into the various levels of mathematics instruction, especially into the topics covered by Intermediate and College Algebra courses. Connecting academic learning to applications in the workplace we call the "real world" can be the motivation for students to become actively involved in the learning process and to become life-long learners as well as productive citizens. Competence in the use of algebra skills has long been recognized as a deciding factor for securing high-tech, high-wage employment. Algebra dropouts simply don't have the opportunities available to those who master those skills.

Lecture only classes which have served us in the recent past, simply do not appeal to a generation of learners who are accustomed to vivid computer animations, interactive video games, and digital quality sound systems. In a real sense, educators are "out-gunned" by the myriad of competition for the attention of learners. It only makes sense to take advantage of the new technologies which are now available to most educators to reconnect with learners on a familiar basis. Thus the use of computer algebra systems, graphing calculators, manipulatives, measurement tools, and interactive systems of instruction and training is encouraged and incorporated into this series of curriculum modules. An attitude of openness toward and implementation of current and future educational technologies can enhance and accelerate learning for students.

While these modules represent a step toward bringing more applied learning techniques into algebra instruction, there are other reform movements in progress to change calculus instruction to also take advantage of new technologies. By building a coherent sequence of reformed mathematics instruction which incorporates applications and hands-on methods, students will be better served and society will benefit.

It is the hope of the authors and the Project Director that YOU will become involved in adapting these modules to your particular needs while making significant improvements to each module. The authors realize that they cannot know the special requirements of your students, but they hope they have started some serious rethinking of how mathematics should be taught.

Harvey Fox, Project Director June 1995

Carl D. Perkins Project # 55170025

Intermediate / College Algebra Applied Methodologies & Accelerated Learning

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Real Number Properties and Operations

APPLIED ALGEBRA CURRICULUM MODULE

Objectives for Real Number Properties and Operations Module

Section 1 : to combine like terms using manipulatives

Section 2 : to add polynomials using manipulatives

Section 3 : to evaluate variable expressions

Section 4 : to subtract polynomials using manipulatives

Section 5 : to demonstrate the distributive property using manipulatives

Section 6 : to multiply polynomials, using manipulatives

Section 7 : to divide polynomials, using manipulatives

Section 8 : to factor quadratics, using manipulatives

Section 9 : to solve linear equations, using manipulatives

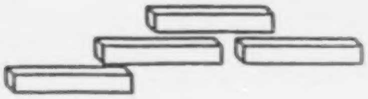
Section 10 : to solve linear equations, using windows

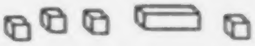
Section 11 : to employ strategies for problem solving

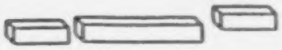
Equipment : Lab Gear , Algebra Tiles , and two-color counters.

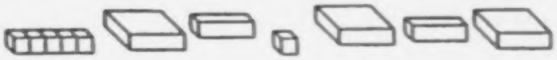
Combining Like Terms

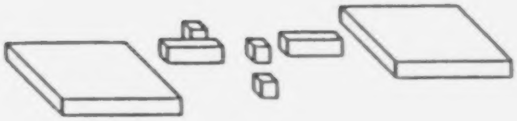
For each example show the figure with your Lab Gear , combine like terms , then write the quantity in simplest form.

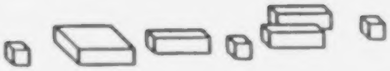
1. 

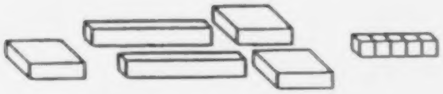
2. 

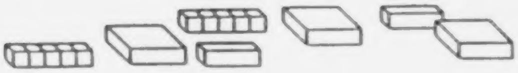
3. 

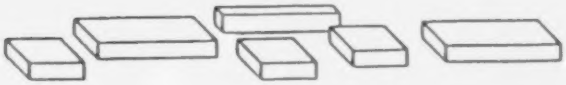
4. 

5. 

6. 

7. 

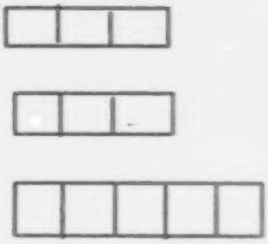
8. 

9. 

Evaluating Variable Expressions

Use the Lab Gear to represent each expression. Sketch it. Draw in the given value for each piece. For example :

$2x + y$ when $x = 3$ and $y = 5$ would look like this



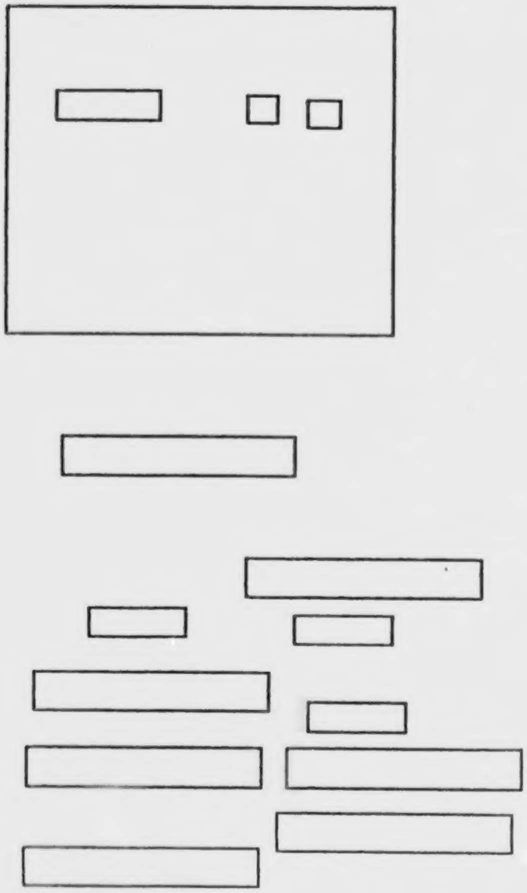
Evaluate each expression for $x = 3$ and $y = 5$.

$5x + 2$ $3y + x + 1$ $xy + 4$ $3x^2$

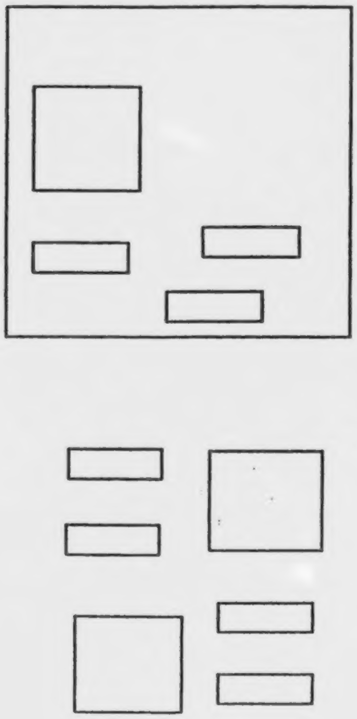
Addition Property

Use the Lab Gear and Minus Box to combine like terms. Remember pieces must be exactly alike to combine. Any piece in the minus box is a negative. When a negative and a positive combine they form a zero pair. All zero pairs should be removed .

$3x + 4y - x + 3y - 2$



$2x^2 + 4x - x^2 - 3x$



Subtraction

Students need to understand the three meanings of the (-) sign: subtraction, negative and opposite.

Subtraction requires two terms. Opposite and negative refer to single terms.

Parentheses are used to distinguish between the sign and the operation.

Whether using the two colored counters or the Lab Gear minus box, students need to understand the definition of subtraction. The " take away " concept gets confusing when the signs are mixed. It is usually easier to use subtraction as the addition of the opposite of the term following the (-) sign. Therefore, 2 - 7 becomes 2 + (- 7). This is followed by forming zero pairs. Again, if students prefer they can use just (+ -) signs rather than the manipulatives. Notice the use of the term , zero pairs , rather than " cancel ". Students who get into the habit of using correct terminology are less likely to confuse operations later.

Students who discover the rule rather than just memorize are more likely to retain it and use it correctly.

Use the two color counters to find the solution. Once you have discovered the rule it will be unnecessary to use the manipulatives.

$2 + 3 =$ (both yellow)

$^{-}2 + {}^{-}3 =$ (both red)

$^{-}2 - 3 =$ (both red)

$^{-}3 + 2 =$ (3 red , 2 yellow)

$^{-}2 + 3 =$ (2 red , 3 yellow)

$^{-}7 + 5 =$

$^{-}5 - 8 =$

$3 - 7 =$

$^{-}14 + 3 =$

$10 + (- 5) =$

The Distributive Property

The distributive property may be illustrated using the Lab Gear. It should be emphasized that the monomial term is being distributed (multiplied) across the polynomial term. This is the same process used in the Area section. Students may prefer to use Algebra Tiles for negative terms. They will eventually progress from the manipulative to the pictorial stage. This could be sketching or the box illustrated in the Area section. The use of negatives, large numbers, decimals, or fractions will force students away for the manipulatives if they have not already begun to wean themselves. Students should have adequate practice with positive terms prior to moving to the negatives, etc.

Using the Lab Gear and the corner piece, build a rectangle with the given dimensions. Find the area of the rectangle.

$3(x + y + 2)$

$x(2x - y + 4)$

$2y(x + y - 3)$

$3x(y + 2)$

$y(x - 5)$

Multiplying Polynomials

Using Algebra Tiles and the corner piece, build a rectangle to represent the problem. You may choose to sketch the pieces. Write your answer in the form Length ● Width = Area.

$(x + 3)(x - 2)$

$(y - 4)(y - 1)$

$(3x + 1)(x + 4)$

$(2y - 3)(2y + 1)$

$(x + 5)(x - 5)$

$(y - 1)^2$

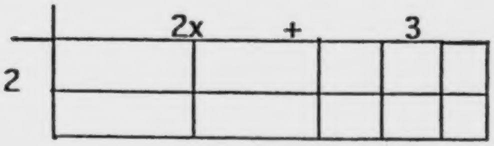
$(x + y)(x + y)$

Dividing Polynomials

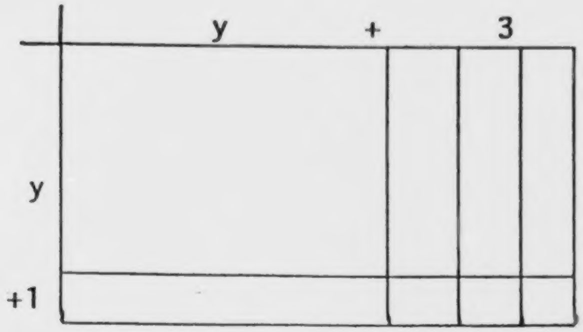
Lab Gear or Algebra Tiles can be used to demonstrate the division of polynomials. Using the Lab Gear and the corner piece, build a rectangle. The divisor should be used as the length and the dividend as the area. This reinforces the visual representation we are used to seeing in a division problem. With the problem set up in this fashion, we are finding the width which is located where we usually write the quotient.

Students should begin with problems without remainders. They can progress to problems with remainders. This will be accomplished be building a rectangle in line with the divisor. Any pieces that don't fit into the rectangle will be the remainder.

$$\frac{4x + 6}{2} = 2x + 3$$



$$\frac{y^2 + 4y + 3}{y + 1} = y + 3$$



Long Division of Polynomials

Use the Lab Gear to show the division. If there are extra pieces that won't fit into the rectangle, they are the remainder.

$\frac{8x + 6}{2}$ $\frac{2x^2 + 6x}{2x}$ $\frac{9x + 3}{3}$

$\frac{4y + 5}{4}$ $\frac{3y^2 + xy + 6y + 2}{y}$ $\frac{y^2 + 3y + 2}{y + 2}$

Factoring Quadratics

Factoring quadratics follows naturally after the division exercise. This activity is designed for the student to discover the factoring process rather than memorize an often meaningless rule.

Using the Lab Gear and the corner piece, build a rectangle. Write your answer in the form Area = Length • Width

$2y + 6$ $\qquad$ $3x + 3y$ $\qquad$ $y^2 + 2y$

Using x^2 and 7 x's with as many yellows as you want, find as many different rectangles as you can. Write your answer in the form Area = Length • Width
What patterns do you see ?

Repeat this using y^2 and 15 yellows with as many y's as you need. What pattern do you see ?

Using Lab Gear and Algebra Tiles

MULTIPLYING
OR
DISTRIBUTING

	GIVEN
GIVEN	FIND

DIVIDING

	FIND
GIVEN	GIVEN

FACTORING

	FIND
FIND	GIVEN

Solving Equations Using Lab Gear

This section is an introduction to solving equations. Students tend to try to memorize rules they don't really understand and don't apply correctly. Through the use of manipulatives they can understand how to solve an equation and develop their own process using inverse operations.

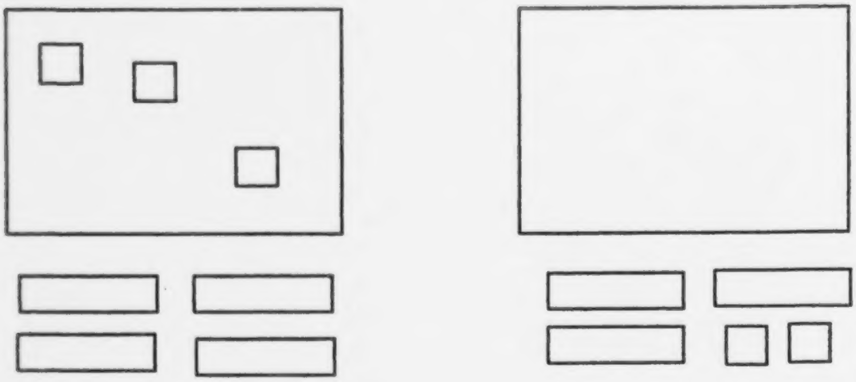
Begin by using Lab Gear and two minus mats to set up the equation. Simplify each side of the equation by removing all zero pairs. Next remove any zero pairs that occur on opposite sides of the equation. Remember they must be both negative or both positive to be a zero pair.

Once this is complete, you must have all variables located on one side of the equation. It is good to have the students get into the habit of locating variables to the left of the equation. This will enable them to input the equation when they use a graphing calculator. If there are any variable terms on the right side of the equation, form matching zero pairs on the left side. This will enable you to form a zero pair across the equal sign. Remember, this is leading to the use of inverse operations. Students need the realize that they can't just move terms around.

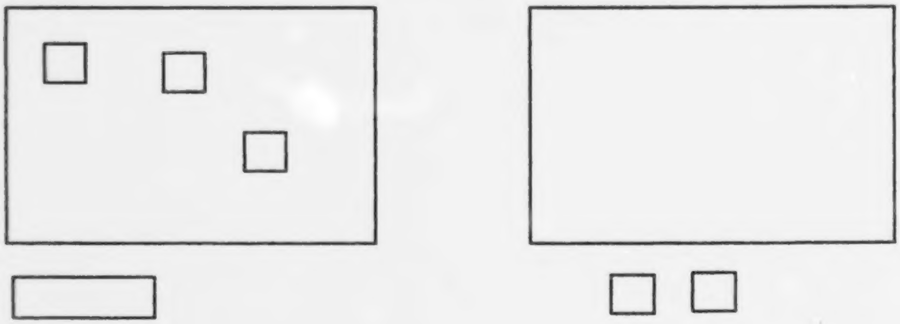
Once all variable terms are located on the left, repeat the operation to locate all numerical terms on the right.

The final step is to divide the numerical terms equally among the variables.

Solve using Lab Gear and the minus mat. $4x - 3 = 3x + 2$

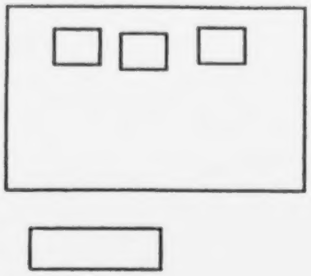


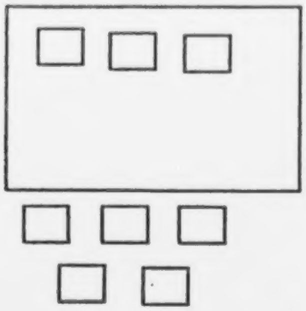
Since there are no vertical zero pairs, you would begin with the x's on opposite sides.



This will leave $x - 3 = 2$.

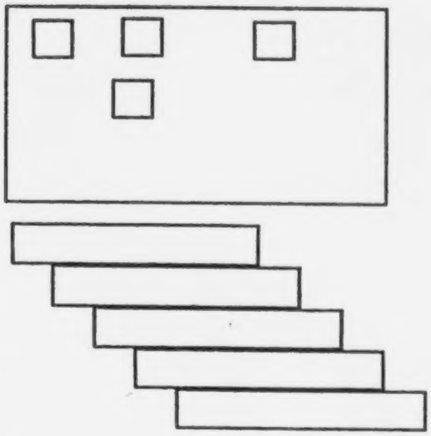
Next you will need to add three negatives and three positives on the right.

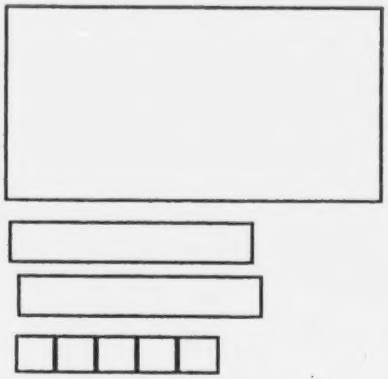




Now you can make three zero pairs in the minus portions. This will leave x = 5 . You should always check by substituting the solution into the original problem. In this case it checks.

Try the process with $5y - 4 = 2y + 5$

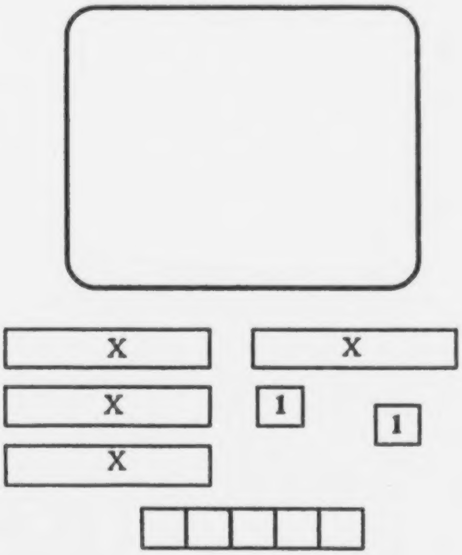


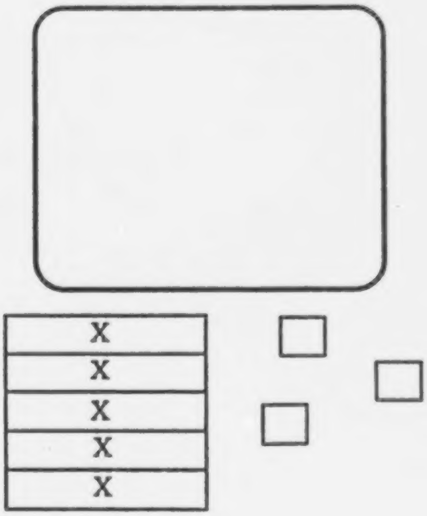


In this problem you will finish by dividing the nine yellows evenly among the 3y's. You may want to use a rubber band to circle your answer.

Linear Equations

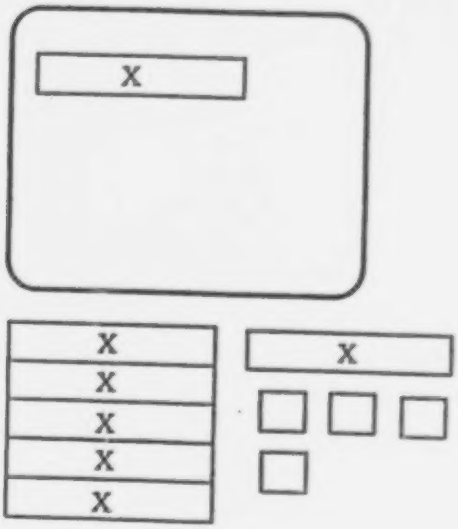
Write and solve each equation.

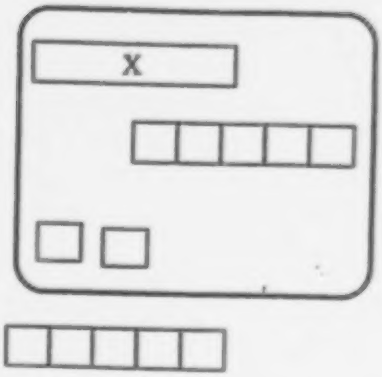




Linear Equations

Write and solve each equation.





Solving Equations Using Algebra Tiles

The procedure for solving equations using Algebra Tiles is very similar to that for solving using Lab Gear. It may actually be easier since you don't need the minus box. The Algebra Tiles express the negatives more directly.

Either of these approaches should make it easier for the student to transfer to using inverse operations. It is important to stress terms such as UNDO, INVERSE OPERATIONS, and ZERO PAIRS. Students tend to confuse operations when they use "cancel " to describe their one size (operation) fits all approach to solving equations. It is easy for them to become confused about needing a zero pair to remove a term or a multiplicative inverse to get a coefficient of one.

Steps to stress:

- simplify both sides of the equation
    - use the distributive property
    - combine like terms
- UNDO any variables on the right side ,using additive inverse
- UNDO any numerals on the left side , using additive inverse
- UNDO the numerical coefficient , using multiplicative inverse.

DO YOU NEED A ZERO PAIR OR A COEFFICIENT OF ONE ?

Solving Equations Using Windows

Students may better understand the process of inverse operations through the use of an activity called WINDOWS or the COVER UP method.

Given the problem $3\square - 4 = 17$

The thinking process would be :

what minus 4 equals 17 ?
21 minus 4 equals 17 , therefore , $3\square$ equals 21
3 times what equals 21 ?
3 times 7 equals 21 , therefore , $\square$ equals 7

Using this procedure, find the value of the $\square$.

$5\square - 1 = 29$

$\square 8 - 9 = 23$

$\frac{48}{\square} + 16 = 4$

$\frac{3\square + 12}{10} = 3$

$\square^2 - 10 = 15$

$4\square - 9 = 23$

Emphasis should be placed on the inverse operation. It may help to use the term UNDO when doing this. The word " cancel " should be avoided. Students tend to use " cancel " when forming a ZERO PAIR and when getting a COEFFICIENT OF ONE . Their confusion is reinforced when both operations are lumped together as " cancel ". The students are more likely to use the correct operation when correct terminology is used.

Find the value of $\square$ in each equation. Write out the steps you used.

$$4 = \frac{8}{\square + 7}$$

$$\frac{3\square}{4} + 5 = 29$$

$$\frac{\square}{10} + 3 = 12$$

$$\frac{{}^{-}2\square + 1}{3} - 3 = 4$$

Strategies for Problem Solving

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Applications

The Alpha Co. needs a cylinder that will hold 200 cu. ft. of material. Jason has located a cylinder on sale for this job. The cylinder has a radius of 3 feet and a height of 7 feet. Using the formula $V = \pi r^2 h$, and 22/7 for π determine if the cylinder Jason found will be adequate.

If Marcia invests $ 5000 in an account paying simple interest of 8.25 % annually, what will her balance be at the end of four years ? Use the formula $A = P + Prt$.

The length of a rectangle is three times its width. If the perimeter of the rectangle is 56 inches, find the dimensions of the rectangle.

James paid $ 18.53 for 17 gallons of gas for his car . If his car averages 21 mpg, what will it cost him to travel to work for a week? The distance each way is 8 miles.

The perimeter of a rectangular garden is 126 feet. The length is twice the width. Find the dimensions of the garden. From the given information, is it possible to determine the maximum area while maintaining the same perimeter ?

Applications

Agriculture : Farming requires a careful balance to maximize the yield per acre. It is critical to find the number of acres in which crops can be grown (area) and the amount of material needed to enclose the field (perimeter). Finding the dimensions of a field can require the use of formulas and unit conversions. It also requires the use of estimation skills.

Example : Jacob plans to enclose his rectangular field with a fence. The area of the field is 625 acres. The perimeter of the field is 5 miles. What are the approximate length and width of the field?

Given that : 1 mile = 5280 feet

1 acre = 43,530 square feet

The first step is to convert to a common measure.

This is a good place to estimate.

5 miles is about 25,000 feet.

625 acres is about 25,000,000 sq. ft.

$$2l + 2w = 25{,}000$$
$$2l = 25{,}000 - 2w$$
$$l = 12{,}500 - w$$

$$lw = 25{,}000{,}000$$
$$(12{,}500 - w)\,w = 25{,}000{,}000$$
$$12{,}5000w - w^2 = 25{,}000{,}000$$
$$w^2 - 12{,}5000\,w + 25{,}000{,}000 = 0$$
$$(w - 10{,}000)(w - 25{,}000) = 0$$
$$w - 10{,}000 = 0 \qquad w - 25{,}000 = 0$$
$$w = 10{,}000 \quad \text{or} \quad w = 25{,}000$$

The dimensions of the field are approximately 10,000 ft. by 25,000 ft.

This problem required: conversion of measurement , estimation , solving an equation to isolate a variable, solving a second equation by substitution , factoring a quadratic and using the zero product property.

Could it have been solved by first isolating a variable in the equation $lw = 25{,}000{,}000$ and then substituting into $2l + 2w = 25{,}000$? Try it.

How would the problem have changed if one side of the field was a river that wasn't to be fenced ?

−

MINUS BOX

+

Problem Solving - Geometric Figures

APPLIED ALGEBRA CURRICULUM MODULE

Objectives for Problem Solving Module

Section 1 : to be able to find the area of a geo-board figure.

Section 2 : to be able to find the perimeter of a figure and to determine the perimeter of other figures using a pattern.

Section 3 : to be able to recognize the difference between the perimeter and the area of figures.

Section 4 : to be able to find the area of a rectangle using manipulatives and pictorial representations.

Section 5 : to be able to find the volume of a rectangular solid using manipulatives.

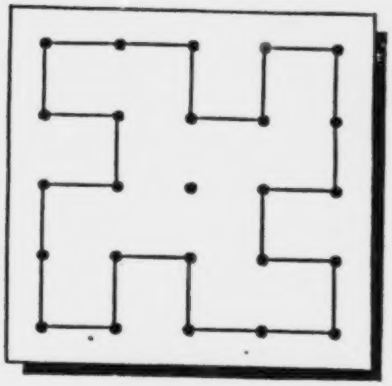
Section 6 : to be able to find surface area .

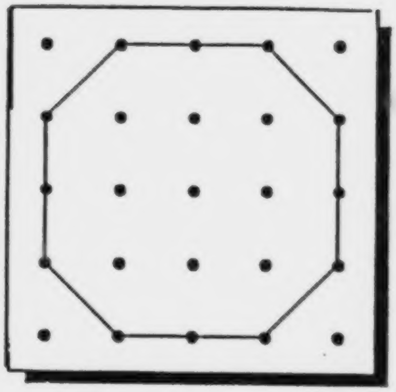
Section 7 : to be able to solve literal equations.

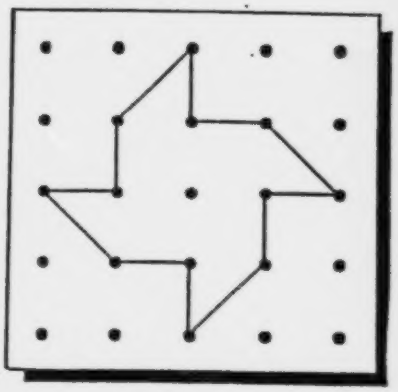
Equipment : Lab Gear , Algebra Tiles , geo- board recording paper , and two-color counters.

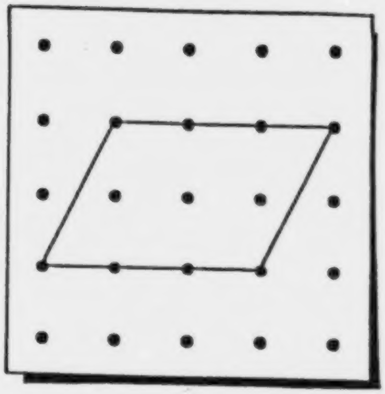
Area

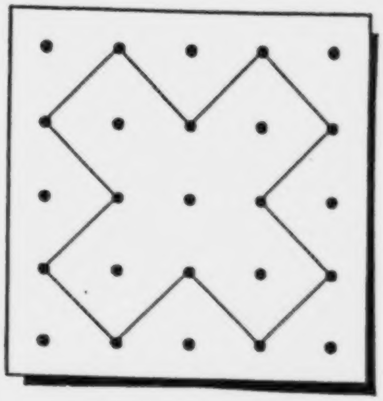
Find the area of each figure by counting the □'s. Each ◺ is half of a □ Some areas are all □'s and some are a mix of □ 's and ◺ 's . You may need to draw a larger rectangle to enclose the area using the diagonal to form a triangle.

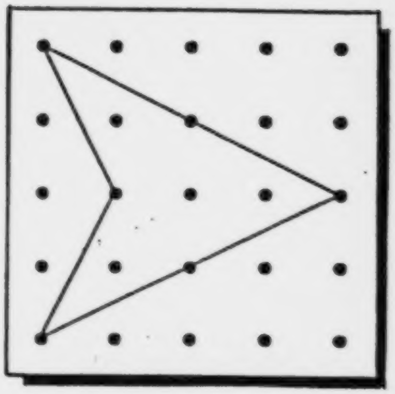


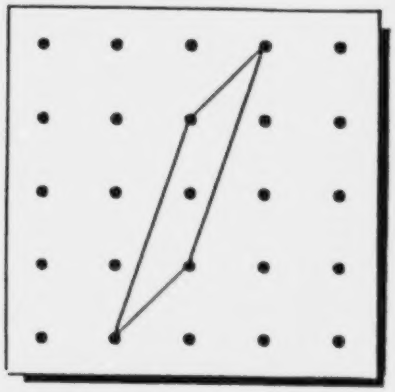


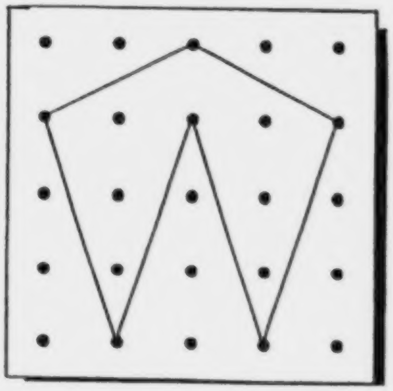


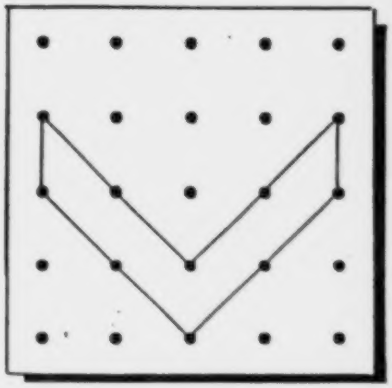








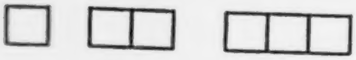




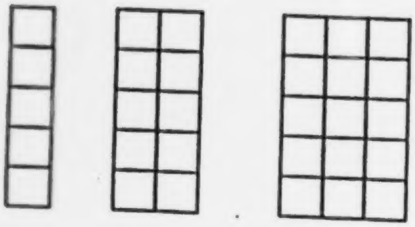
Perimeter

Look at each sequence. Write the perimeters of the figures given. Think about how the pattern continues. Use the pattern to determine the perimeter of the fourth, the tenth, and the nth terms.

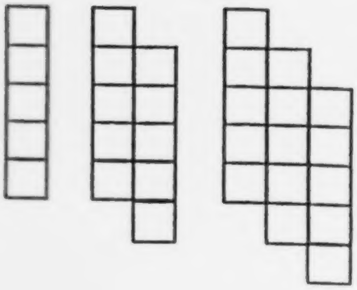
1.



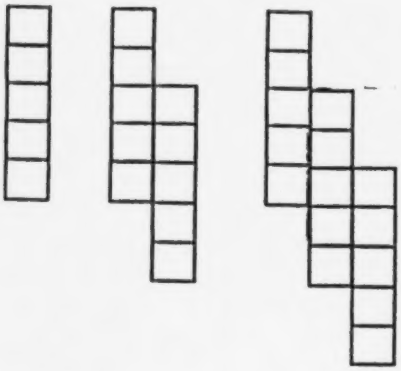
2.



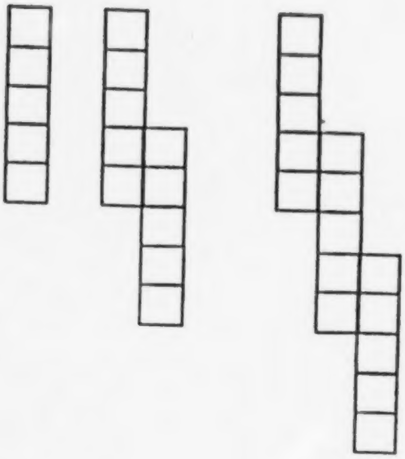
3.



4.

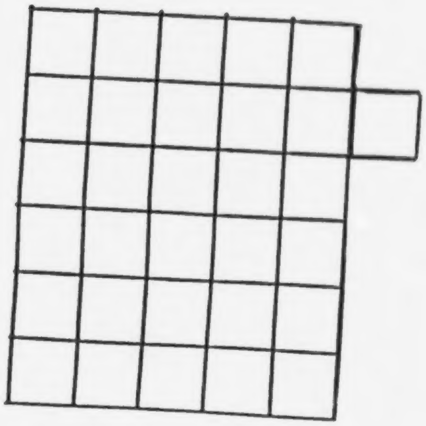


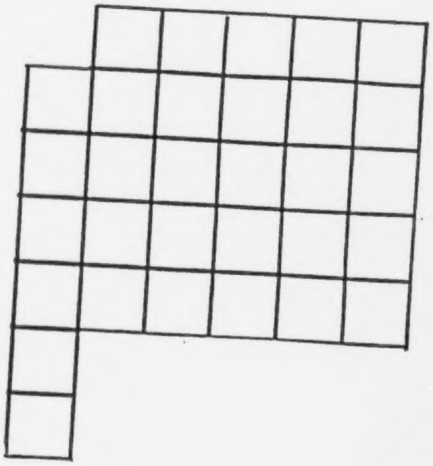
5.

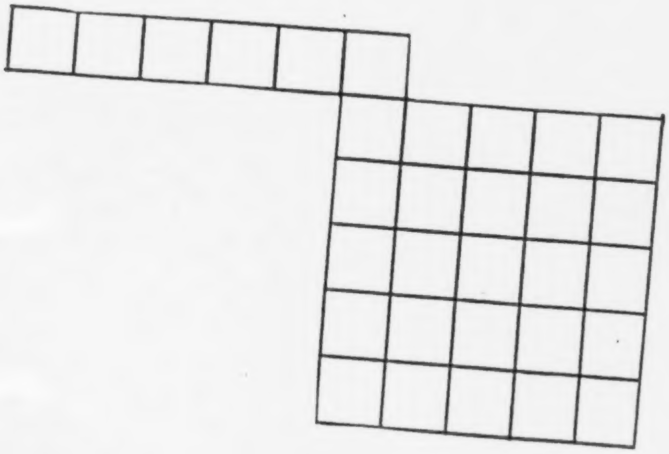


Perimeter and Area

Each of these figures represents the area formed by Lab Gear units pieces. The areas of all the figures are equivalent but the perimeters are different. Find the area and the perimeter of each figure.

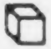






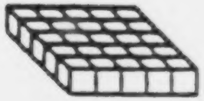
The Algebra Lab Gear

These are the blocks that make up a set of Algebra Lab Gear.

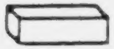


1

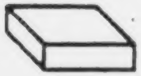
5



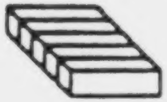
25



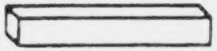
x



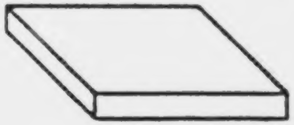
x^2



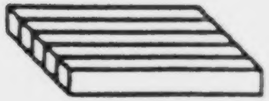
$5x$



y



y^2



$5y$

xy

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Area

Use the Lab Gear and corner piece to show each product. Write your answer in the form Length times Width = Area. You may find it helpful to sketch the Gear.

$3(x+2)$

$x(x+5)$

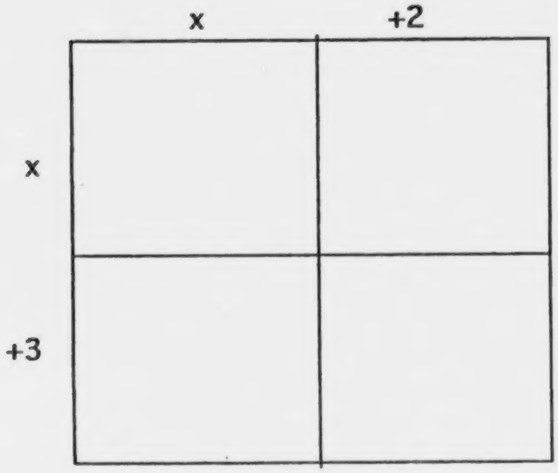
$y(y+4)$

$(x+2)(x+3)$

$(y+1)(y+6)$

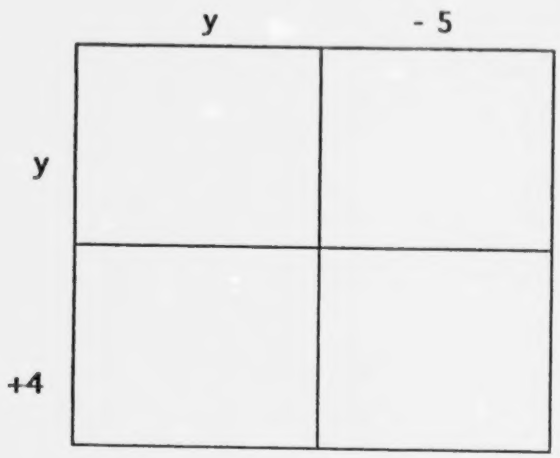
$(x+4)(x+2)$

Students will eventually tire of the Lab Gear and the sketching. They may feel more comfortable using a pictorial algebraic configuration. Each of the inner rectangles is the product of the edges. The like terms are combined by adding. This also reinforces the concept of area as the product of length and width. Students frequently confuse that with perimeter.



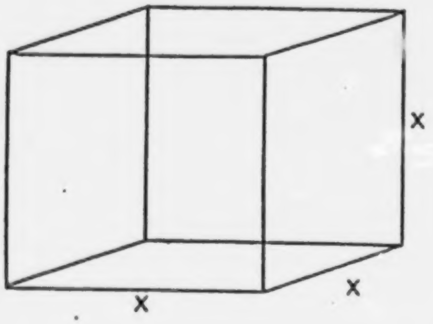
This is a more concrete representation than the FOIL method we are used to seeing. It may be helpful to present this in combination with the FOIL method for reinforcement.

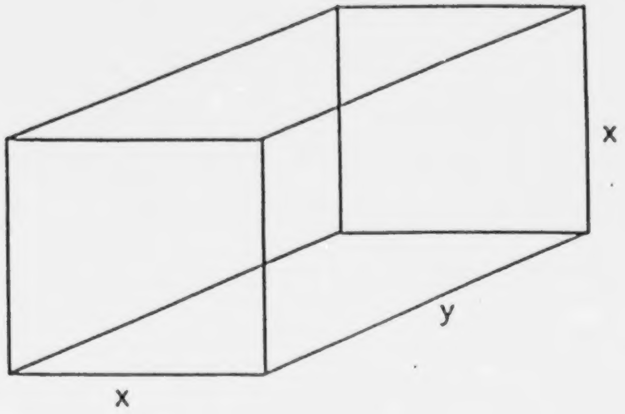
This format is especially helpful when dealing with negatives. Students need to remember the rules for multiplication of negatives when finding the area. They also need to remember the rules for addition of signed numbers when combining like terms. If students have used the Lab Gear first, it will be easier for them to understand that only like terms can be combined.

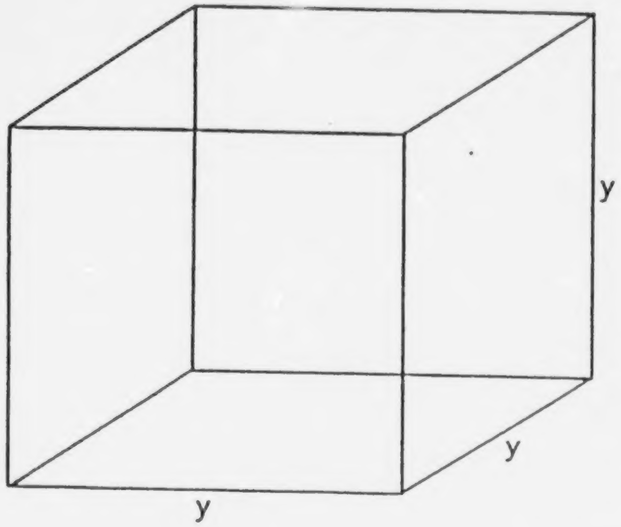


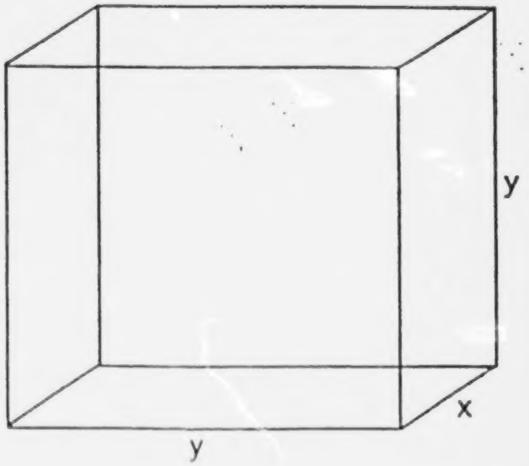
This representation also has the advantage of adapting to any polynomial product. You simply adjust the number of sections to accommodate the terms of the polynomials. It can also be used for polynomials of any degree, but I suggest students have adequate practice with second degree polynomials before advancing.

3 - D Lab Gear









Volume

Use the Lab Gear and the corner piece to build a rectangular box. Write your answer in the form Volume = Length ● Width ● Height

$xy^2 + 2y^2$

$x^2y + xy^2 + xy + y^2$

$y^3 + y^2 + xy^2$

$xy^2 + 2xy + y$

$xy^2 + x^2y + 3xy$

Surface Area

Using the 3-D Lab Gear, find the surface area of each piece. Remember that each piece has six surfaces. Identify each surface, then combine like terms.

x^3 y^3 xy^2 x^2y

Find the surface area of the remaining Lab Gear pieces. You may find it helpful to sketch the pieces.

1 5 25

x $5x$ $25x$

y $5y$ $25y$

xy

Solving Literal Equations

The ability to solve literal equations is very helpful when using formulas. Given the formula Distance = Rate ● Time , why should we memorize three formulas, solving for Distance, Rate , and Time when we can memorize the original and solve for the other two ?

If we focus on the desired variable, we can use the process of UNDO or inverse operations to isolate the variable. We start with a single formula and UNDO until we isolate the variable.

The formula $I = Prt$ gives the interest I corresponding to an initial deposit of P dollars, an annual percentage rate of r, and a time of t years. To solve for r , start with $I = Prt$. Factor out the r . Since P and T are multiplying the r , use the inverse operation - division . The result will be $r = \frac{I}{Pt}$

The formula for the balance of the account would be $A = P + Prt$. Factor out the common P . This gives $A = P(1 + rt)$. To solve for the P , we need to realize that P is multiplying the quantity (1+ rt). The inverse of multiplication is division, therefore we divide by the quantity (1 + rt). The result is $P = \frac{A}{1 + rt}$. Using one formula we can find any term .

48

GRAPHING SKILLS
APPLIED ALGEBRA CURRICULUM MODULE

***Performance Objective:* Students will be able to construct graphs given coordinate pairs of data and will be able to construct, read, and interpret curves of best fit given plotted data pairs.**

Construction of Graphs

Investigation/Demonstration:
In the first investigation we will consider computer-generated graphs on a graphing calculator.

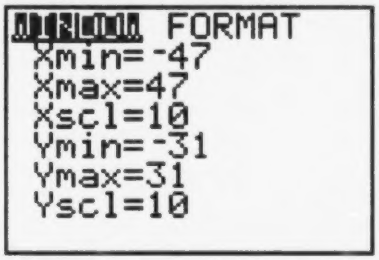
Union Package Service will ship small jewelry boxes anywhere in the country for a $3.00 pickup charge plus $0.75 per jewelry box while Rover Package Movers will ship jewelry boxes anywhere in the country for a $4.00 pickup charge and $0.65 per jewelry box.

Write an algebraic expression for the cost of shipping of T jewelry boxes through Union Package Service.

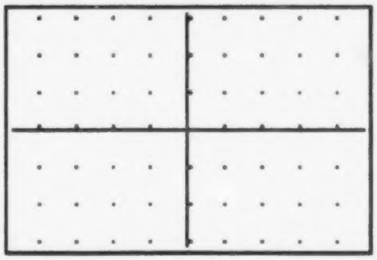
Write an algebraic expression for the cost of shipping of T jewelry boxes through Rover Package Movers.

Write an algebraic expression of the form:
Cost of shipping M jewelry boxes by UPS = Cost of shipping M jewelry boxes by RPM.

Set the viewing window on the graphing calculator to the window settings shown here



WINDOW FORMAT
Xmin=-47
Xmax=47
Xscl=10
Ymin=-31
Ymax=31
Yscl=10



and enter the algebraic expression for the left side of the algebraic expression as Y1 on the **Y=** screen. Enter the algebraic expression for the right side of the algebraic expression as Y2 on the **Y=** screen. Use the **TRACE** key to find a value for X where the Y-coordinates are equal. You can use the cursor keys ‘up’ and ‘down’ to toggle from Y1 to Y2 as you trace. At this point the value for X = _______ and the value for Y = _________

What does the solution tell us about the shipping charges for the two companies?

On the TI-82, use the **TblSet** to build a table of values for X = 0 to 15 in steps of 1 and then press TABLE to see a comparison of costs generated by the expressions in Y1 and Y2. For what value of X will the costs from the two companies be the same?

How does the answer obtained from the tables compare to the answer obtained by tracing on the graphs?

What is the cost for shipping 75 jewelry boxes through UPS? through RPM?

What is the minimum number of jewelry boxes that could be shipped by UPS before a savings of $10 in shipping costs over RPM could be realized?

Activity

A one-inch wide strip of elastic is used to support some small weights. Show how the elastic is being stretched by increasing sizes of weights and you begin to discover a pattern that can occur in linear model equations.

Create your own data sets or plot the data from the table on the graph below. Each data pair represents data from a test of a series of weights, in ounces, being attached to an elastic band. The amount of stretch, in inches, is the vertical axis. The weights are represented on the horizontal axis.

Weights (in ounces)	0	1	2	3	4	5	6	7	8	9	10
Stretch (in inches)	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0

The rate of change of the length of the elastic band as a function of the weights and is given by : $\frac{\Delta \text{Length}}{\Delta \text{Weight}} = \frac{\text{Vertical Change}}{\text{Horizontal Change}}$

- What is the rate of change for the stretch in the elastic band when the change in weight is 2 ounces? 4 ounces? 8 ounces?

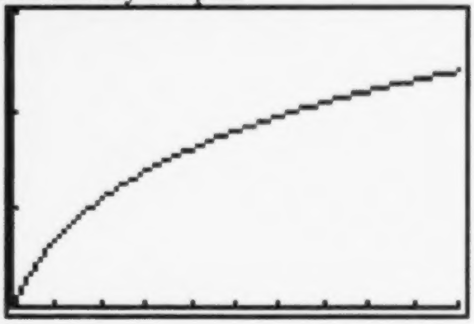
- How is this rate of change shown in the graph above?

- The length of the elastic band with no weight is 4 inches. What is the length of the elastic band when the weight is 2 ounces? 4 ounces? 8 ounces?

- Use the graph or the equation to predict the stretch lengths of the elastic if the weights attached total 7.5 ounces? 24 ounces? 1.75 ounces?

Reading and Interpreting Graphs

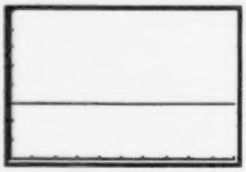
Investigation/Demonstration:
Suppose the graph below is such that the horizontal axis represents *time* in hours from 9 a.m. until 7 p.m. on a given day and the vertical axis represents *anxiety level* for someone who is going on a “blind” date on this day at 7 p.m.



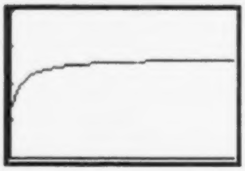
Describe another possible situation that the graph above might represent. Be sure to describe both the *horizontal* and *vertical* axis.

Which graph below best represents the relationship between *speed* of a distance runner (vertical axis) and *time* elapsed (horizontal axis) from the starting line to the finish line?

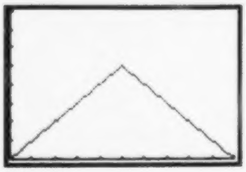
(a)



(b)



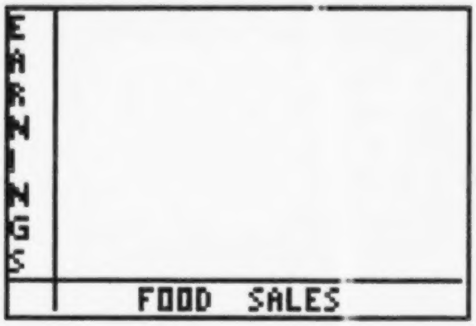
(c)



Activity

Miguel and June each work one six-hour shift at China Star Restaurant on weekends. Miguel busses trays from the tables and June waits tables. The restaurant has a policy that a 15% tip is included in all checks for customers. Waiters receive $15 per shift plus 10% in tips while bussers receive $25 per shift and 5% in tips.

- Write an equation to model the wages earned (before taxes) by Miguel in one weekend by working as a busser.
- Write an equation to model the wages earned (before taxes) by June in one weekend by working as a waiter.
- Graph both equations in the same viewing window and use the TRACE key to find the coordinates of the intersection point.
- Sketch the graph below.



- What does the X-coordinate represent for this problem situation?
- What does the Y-coordinate represent?
- Suppose that business at the restaurant was unusually slow one weekend and the restaurant had very few customers during the shifts worked by Miguel and June. Whose earnings would probably be lower?
- Suppose that business was very brisk just after the Thanksgiving holidays and that $1200 in food sales were recorded during the shift worked together by Miguel and June. Which one of them would earn the most money during that shift?
- What is the difference in the earnings by Miguel and June during that shift?

Scatterplots

Investigation/Demonstration:

A scatter plot is a visual way for testing whether or not two quantities are related.

Measure and record the body height and the shoe length of each student in your class.

Prepare a scatter plot of the data using a graphing calculator. If you are using a TI-82 follow the steps below:

ON THE TI-82...

1. Press the **MODE** key and select **Function** mode.

2. Press **Y=** and be sure that all functions are cleared.

3. Press **2nd Y=** and from the **STAT PLOTS** menu select **4:Plots Off.** Press the **ENTER** key until the word "Done" appears on the HOME screen.

4. Press the **STAT** key and the **EDIT** menu will appear. From this menu, select **4:ClrList** by pressing the **4** key. The ClrList will be "pasted" to the HOME screen. Now press **2nd L_1**, a comma, and **2nd L_2**. (The comma key is on the sixth row, the second key) Now press **ENTER** and the the ClrList will be Done.

5. Press the **STAT** key and the **EDIT** menu will appear. From this menu, select **1:Edit...** by pressing **ENTER**.

6. Enter the *height* data one at a time in list **L_1**. After the last data element has been entered in **L_1**, press the right cursor arrow and enter the *shoe length* data one at a time in **L_2**. When all data have been entered, press **2nd QUIT**.

7. Press the **WINDOW** key and enter your choices for the window settings one at a time.

8. Press **2nd Y=** and the menu for Plot1 will appear as in the box on the left. Use the selections from the the box and press **GRAPH** to see the scatter plot.

Fill in your selections for these window settings that you used to produce the scatterplot:

Xmin = ______ Ymin = ______
Xmax = ______ Ymax = ______
Xscl = ______ Yscl = ______

Does the scatter plot seem to indicate that a relationship exists between *height* and *length of shoe*? Explain.

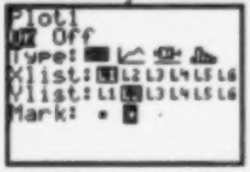
Suppose that you found a shoe-print in the sand that measured 15 inches long, how tall do you think the owner of the shoe might be?

Activity

The graphing calculator can be used to analyze two-variable data in several ways. The data will be represented in a scatterplot and then a trend line, sometimes called a line of best fit, will be drawn. The methods for entering, editing, analyzing, and displaying data on the TI-82 are given in this table:

On the TI-82...

1. Press the **Y=** key and clear all functions from this screen. Press the **STAT** key and the **EDIT** menu will appear.
2. Select **4:ClrList** by pressing '4' and then press **2nd L1**, followed by a comma (press the **,** key, and then press **2nd L2**.
3. Press the **ENTER** key and the word 'Done' will appear on the right of your screen to confirm that you have cleared all data.
4. Press the **STAT** key and select **1:Edit...** by pressing the **ENTER** key.
5. Lists **L1**, **L2**, and **L3** will appear on your screen. Enter the x-values of your data pairs in the column labeled **L1** and y-values in **L2**.
6. When all of the data pairs have been entered, press **2nd QUIT** to return to the HOME screen.
7. Press the **WINDOW** key and enter values for setting an appropriate viewing window to display the data.
8. Press **2nd STAT PLOT** and from the **STAT PLOT** menu, select **1:Plot1...** by pressing the **ENTER** key. Make selections from this menu like those in this frame.



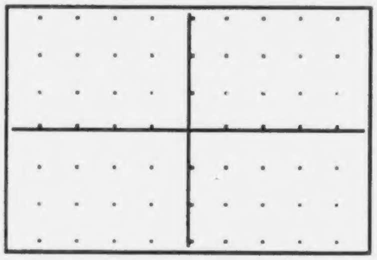
Plot1
On Off
Type: ▪ ⊾ ⊡ ⫠
Xlist: L1 L2 L3 L4 L5 L6
Ylist: L1 L2 L3 L4 L5 L6
Mark: ▪ □ ·

9. Press **GRAPH** key to see the scatterplot.
10. Press **STAT** and move the cursor to the **CALC** menu.
11. Select **9:LinReg(a+bx)** by pressing '9'. Press **2nd L1**, followed by a comma, and then **2nd L2**. Press the **ENTER** key to see the coefficients of the linear regression model.
12. Press the **Y=** key to prepare to receive the linear regression equations. Press the **VARS** key, then select **5:Statistics...** and move the cursor to the right to highlight **LR**. From this menu select **7:RegEQ** and the regression equation will appear on Y1.
13. Press **GRAPH** to see the trend line.

- Enter the data from the table below into the graphing calculator following the directions as given above.
- Find the linear regression equation. Use two decimal places for accuracy when you record your answers. Set an appropriate viewing window.
- Make a scatter plot of the data and graph the linear regression on the same screen.

x	y
−13	49.9
−11	28.5
−10	36.9
−8	18.3
−4	11.8
3	7.3
5	−14.1
8	−10.3
11	−30.1
15	−23.1

- Sketch the graph in this window.



Curve-Fitting

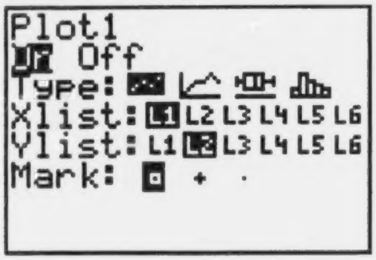
Investigation/Demonstration:

Some relationships are based on geometric and numerical patterns rather than statistical data.

Complete the table below:

Term	1	2	3	4	5	6	7	8	9	10	11	12
Value	1	4	9	16	25						121	

Using a TI-82 graphing calculator, enter the data for "term" in list **L$_1$** and the data for "value" in list **L$_2$**. Select an appropriate viewing window and use the settings given in the box below for **Plot1.**



Plot1
On Off
Type:
Xlist: L1 L2 L3 L4 L5 L6
Ylist: L1 L2 L3 L4 L5 L6
Mark: □ + ·

Use the **TRACE** key on your graphing calculator and trace from point to point on your screen. How do the X-values and Y-values on the read-out on your screen compare to the values in your table?

Activity

A ball is dropped from the rafters in a large gymnasium onto the hardwood floor 45 feet below. Assume that the basketball will always rebound to approximately half the distance of the fall.

- To what height will the ball rebound after the 3rd bounce?
- How many times will the ball be at least 4 feet above the hardwood?
- When will the ball stop bouncing?
- Explain.
- Use a graphing calculator to produce values which show the expected rebound height for the basketball for the first ten bounces.
- Use these values to complete the table below.

Number of Bounce	0	1	2	3	4	5	6	7	8	9	10
Height of Rebound	45 ft										

Histograms

Investigation/Demonstration:

Bar graphs can be represented on the graphing calculator by entering data, by setting a viewing window, and by selecting the histogram. Instructions for completing a bar graph are given below:

On the TI-82...

1. Press the **Y=** key and clear all functions from this screen. Press the **STAT** key and the **EDIT** menu will appear.

2. Select **4:ClrList** by pressing '4' and then press **2nd L1**, followed by a comma (press the **,** key, and then press **2nd L2**.

3. Press the **ENTER** key and the word 'Done' will appear on the right of your screen to confirm that you have cleared all data.

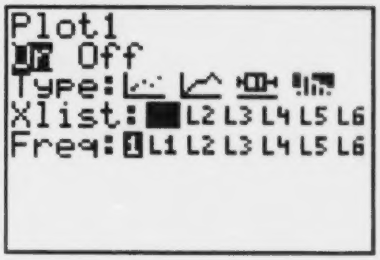
4. Press the **STAT** key and select **1:Edit...** by pressing the **ENTER** key.

5. Lists **L1**, **L2**, and **L3** will appear on your screen. Enter the x-values of your data pairs in the column labeled **L1** and enter the y-values in **L2**.

6. When all of the data pairs have been entered, press **2nd QUIT** to return to the HOME screen.

7. Press the **WINDOW** key and enter values for setting an appropriate viewing window.

8. Press **2nd STAT PLOT** and from the **STAT PLOT** menu, select **1:Plot1...** by pressing the **ENTER** key. Make selections from this menu like those in this frame.



Plot1
On Off
Type:
Xlist: L1 L2 L3 L4 L5 L6
Freq: 1 L1 L2 L3 L4 L5 L6

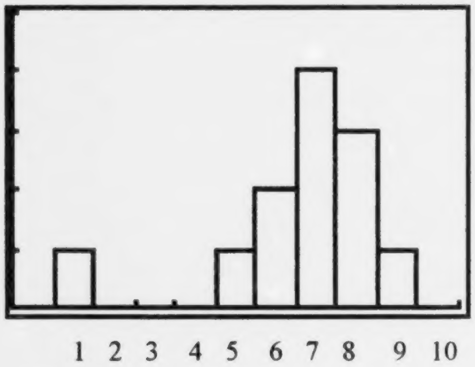
9. Press the GRAPH key to see the histogram.

Have each student in your class count the amount of change they have in their purses or pockets. Ask each student to say aloud the amount of change they have in their purses or pockets and record them in a table.

Use the procedure given above for constructing a histogram on a graphing calculator. Enter the pocket change data in your calculator and construct the histogram. Sketch the histogram in the window below:

Activity

Consider the bar graph given below which shows the ratings by a panel of twelve judges for a contestant in a talent show competition. The contestants are judged on a ten-point scale with 1 being the lowest rating and 10 the highest.



- What does the horizontal axis represent in this bar graph?

- How many judges rated this contestant with a '7' or better?

- What was the average rating for this contestant?

- How did you arrive at your answer?

- Is there any evidence of bias against this contestant based on the ratings given in the bar graph? Explain.

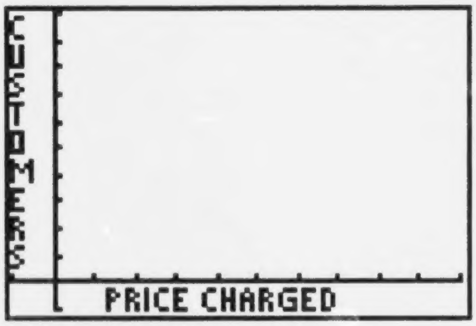
- Could the graph above be used to mislead someone about this contestant's ratings?
- Based solely on the ratings given in the bar graph above, how would you rate this contestant? Why?

Evaluation Instrument

A market research firm has completed a survey of local tanning booth operations. People were asked how much they would pay to take a session in a tanning booth. Based on those results the number of potential customers each day at several typical prices are given in the table.

Price Charged per Visit	10.00	15.00	20.00	22.50	25.00	27.50	30.00	33.00	35.00	40.00
Customers Per Day	42	35	31	29	28	24	23	17	14	10

1. Find a line-of-best-fit, or trend line, for the data in the table. Do a scatterplot of the data in the same viewing window as the trend line. Sketch the line and the scatterplot in the frame given below.



2. What trend do you observe in the relation between price charged per visit to the tanning booth and customers per day as shown in the plot?

3. How will the number of customers probably change as the price is increased higher and higher?

4. Find the equation of the regression line.

5. Use the linear model that you found on your calculator and use the TRACE key to estimate the number of customers that you might expect if the price is set at $15
at $25 at $35

6. Suppose you found that the tanning booth business averaged 32 customers per day for a period of two weeks. What is your estimate of the price that had been charged per visit?

Materials required for teaching this unit:

TI-82 Graphing Calculators
TI-82 Overhead Viewscreen
Centimeter Grid Paper
Elastic Strips
1 ounce, 2 ounce, and 3 ounce fishing weights
Rubber Bands
Eyelet
Measuring Tapes
Centimeter and inch rulers
Colored Marking Pens

EXPONENTS AND ROOTS

APPLIED ALGEBRA CURRICULUM MODULE

PERFORMANCE OBJECTIVE:

The student will correctly solve problems with exponents and roots at the 70% mastery level as demonstrated on a posttest.

BACKGROUND INFORMATION:

When discussing the history of exponents and roots, it is important to realize that before the invention of computers and calculators, common logarithms were used to perform arithmetic calculations. Values of common logarithms were found in tables, and it was common for most science textbooks to include a table of common logarithms to help with the computations in the text. Book-length tables of logarithms, carried out to many decimal places, were considered to be standard equipment for anyone who needed to execute lengthy scientific calculations involving powers.

Common logarithms were invented by the English mathematician Henry Briggs in the 17th century. In fact, in some old books, common logarithms were often called Briggsian logarithms. Although common logarithms are an anachronism for computational purposes today, it is impossible to overemphasize the great advance in calculation that they afforded to scientists of the 17th through the 19th centuries. Important calculations in astronomy, physics, and chemistry became possible only after logarithmic tables became available.

These tables were so important to calculation that when the Works Progress Administration (WPA) was looking for jobs for unemployed scientists and mathematicians during the Great Depression, they commissioned a new set of logarithm tables, carried out to 14 decimal places.

Exponential scales are commonly used in acoustics (dB scales), electronics (VU scales), and chemistry (pH scales).

DEMONSTRATION:

The instructor will demonstrate one historical use of exponents and roots by demonstrating the use of a slide rule and by allowing students to solve simple problems with slide rules. (Since slide rules are no longer available in stores, this module includes a copy master for making paper slide rule simulators for student use. "Real" slide rules would be preferable if they can be located.) The instructor should explain that "Once upon a time before

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calculators there were slide rules." Slide rules have exponential scales (scales C and D) which can perform multiplication and division operations to 3-significant digits. First show the students how two meter sticks (or rulers) can be use to add or subtract two numbers by sliding the meter sticks to align given numbers. Then use the exponential scales (C and D) on the slide rules to "add" and "subtract" the exponents of given numbers to perform multiplication and division of those given numbers. (Use the handout provided with the copy master of the slide rules which describes these procedures if you are not familiar with the use of slide rules.) **Ask students to conjecture a pattern or rule which relates the meter stick and slide rule operations**.

ACTIVITIES:

"Odd Oscillations" is the initial activity which poses a real problem which involves the use of roots (or fractional exponents). Students should be allowed to attempt solutions to the problem without any formal instruction in exponents and roots. At some point, students may indicate a need for assistance which is when the instructional component of the properties of exponents and roots can be introduced. (Once the instruction has been completed, don't forget to return to the "Odd Oscillations" problem to "find" the solution.)
There will also be many "small/short" activities which will be grouped into two larger groups of activities. These two groups are A) verifying properties of exponents and roots with a calculator, and B) solving workplace- related / technical problems dealing with exponents and roots.

A) Verifying properties of exponents and roots with a calculator
The instructor will demonstrate the properties of exponents and roots by having the students verify the rules by working problems with a calculator. This activity will both improve the students' calculator skills as well as help the students to see that the properties work and are not just made up by instructors, textbook authors, etc. (A handout is attached.)

B) Solving workplace-related / technical problems dealing with exponents and roots
The instructor will assist small collaborative groups of students as they work through a series of problems dealing with such topics as electricity, wastewater technology, land value, manufacturing technology, etc. This will help the students to see a real-world use of exponents and roots. (A handout is attached)

PROBLEM: ODD OSCILLATIONS

You have inherited an old electronic keyboard instrument which still works, but it is badly out of tune. You have discovered that there are 12 tuning adjusters, one for each note in an octave. If you tune one octave of keys all of the other octaves will also be tuned. Since you do not have perfect pitch hearing, you decide to use an electronic frequency counter to perfectly tune all 12 notes. You have learned that middle A has a frequency of 440 Hz and that A' which is one octave higher has a frequency of 880 Hz. To what frequencies should you tune the notes between A and A'? (NOTE: Equally tempered scale semitone frequencies change by a constant multiplier of the previous frequency.)

MULTIPLIER = ______

NOTE NAMES	**FREQUENCIES (Hz)**
A	440.00
A#	______
B	______
C	______
C#	______
D	______
D#	______
E	______
F	______
F#	______
G	______
G#	______
A'	880.00

SOLUTION: ODD OSCILLATIONS

NOTE NAMES	FREQUENCIES (Hz)	MULTIPLIER
A	440.00	$\sqrt[12]{2}=1.05946$
A#	466.16	
B	493.88	
C	523.25	
C#	554.37	
D	587.33	
D#	622.25	
E	659.26	
F	698.46	
F#	739.99	
G	783.99	
G#	830.61	
A'	880.00	

Verification of Properties Using a Calculator

Let **a** and **b** be real numbers, variables, or algebraic expressions such that the indicated roots are real numbers, and let **m** and **n** be positive integers. Assume all denominators and bases are nonzero.

Properties of Exponents	Example
1. $a^m a^n = a^{(m+n)}$	$3^2 \bullet 3^4 = 729$ $3^{(2+4)} = 729$
2. $\frac{a^m}{a^n} = a^{(m-n)}$	$\frac{2^7}{2^4} = 8 = 2^3$ $2^{(7-4)} = 8 = 2^3$
3. $a^{-n} = 1/a^n = (1/a)^n$	$2^{-3} = 0.125$ $1/2^3 = 0.125$ $(1/2)^3 = 0.125$
4. $a^0 = 1,\ a \neq 0$	$999^0 = 1$
5. $(ab)^m = a^m b^m$	$(5 \bullet 2)^3 = 1000$ $5^3 2^3 = 1000$
6. $(a^m)^n = a^{mn}$	$(4^2)^3 = 4096$ $4^{2 \bullet 3} = 4096$
7. $(a/b)^m = a^m/b^m$	$(3/5)^2 = 0.36$ $3^2/5^2 = 0.36$

Properties of Roots	Examples
8. $\sqrt[n]{a^m} = (\sqrt[n]{a})^m$	$\sqrt[3]{8^2} = \sqrt[3]{64} = 4$ $\quad (\sqrt[3]{8})^2 = 2^2 = 4$
9. $\sqrt[n]{a} * \sqrt[n]{b} = \sqrt[n]{ab}$	$\sqrt{32} * \sqrt{2} = 8$ $\quad \sqrt{(32*2)} = 8$
10. $(\sqrt[n]{a}) / (\sqrt[n]{b}) = \sqrt[n]{(a/b)}$	$(\sqrt{50}) / (\sqrt{2}) = 5$ $\quad \sqrt{(50/2)} = 5$
11. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$	$\sqrt[3]{\sqrt{64}} = 2$ $\quad \sqrt[6]{64} = 2$
12. $(\sqrt[n]{a})^n = a$	$(\sqrt{7})^2 = 7$

WORKPLACE / TECHNICAL PROBLEMS

1. When estimating the forest-land value, a formula such as shown below is often used.

$$V=\frac{N}{(1+I)^{t}-1}$$

where **V** is the land expectation value in dollars per acre ($/A),

N is the net income received at rotation age ($),

I is the interest rate expressed as a decimal, and

t is the length of rotation in years.

Determine the land expectation value for a pine forest based on a rotation of 60 years, an interest rate of 5%, and a net income of $210 at rotation age.

2. The spark plugs in most automotive engines must provide a spark for ev6ry two revolutions of the engine. While driving at nominal speeds, the engine may be running at about 2500 revolutions per minute (rpm).

   a. If an average speed of 1 mile per minute is assumed for an annual mileage of 10,000 miles, approximately how many minutes is the car driven during the year?

   b. At 2400 revolutions per minute, or 1200 sparks per minute, during the year's driving, about how many times does the spark fire during the year? (Express your answer in scientific notation.)

3. For relatively low temperatures, a thermocouple made with lead and gold wires produces 2.90 microvolts for each degree Celsius (using 0°C as the reference).

   a. Express the voltage as volts per degree Celsius in scientific notation.

   b. What voltage would you expect from a thermocouple experiencing a temperature of 15°C? (Express your answer in scientific notation.)

4. Radio and television frequencies are given in hertz (Hz), or cycles per second (cps). Listed below are some frequencies for other common forms of electromagnetism. Use the prefixes to convert each frequency to scientific notation with units of Hz.

Type broadcast	Frequency
Household electricity	60 Hz
AM radio	1080 kHz (kilohertz)
Short-wave radio	10 MHz (megahertz)
FM radio	102 MHz (megahertz)
Radar	8 GHz (gigahertz)
Microwave communication	12 GHz (gigahertz)
Visible light	400 THz (terahertz)

5. A common measure applied to solutions is its pH-a measure of the hydrogen-ion activity of a solution. The hydrogen activity of pure water at 25°C is 0.0000001 moles per liter. A highly acidic solution has 1.0 mole per liter of hydrogen activity, while a highly basic solution has an activity of 0.00000000000001 moles per liter.

   a. Express each of the three hydgrogen activities given above in scientific notation.

   b. The pH value of a solution is simply the exponent of ten of the measure of its hydrogen activity. What is the pH value associated with the highly acidic solution above? with the pure water? with the highly basic solution?

6. A computer is advertised as having a processing speed of "11 mips," or 11 million instructions per second.

   a. Express this speed in scientific notation.

   b. On the average, how long does it take to process each instruction at such a speed?

   c. How many "nanoseconds" is this?

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7. A wire-wound resistor with a resistance of 1 ohm (1 Ω) is needed. You have a supply of 8-gauge, 24-gauge, and 36-gauge copper wire that has a resistivity of 1.72×10^{-8} Ω•m. The cross-sectional area of the 8-gauge wire is 8.367×10^{-6} m^2, of the 24-gauge wire is 2.048×10^{-7} m^2 , and of the 36-gauge is 1.267×10^{-8} m^2.

   a. Compute the resistance per meter of each gauge wire by dividing the resistivity by its cross-sectional area.

   b. Use the resistance per meter computed in Part a to determine what length of wire would be needed to obtain the desired resistance of 1 Ω, for each wire gauge.

8. The alternating current reactance of a circuit, X_L, is given in ohms (Ω) by the formula

$$X_L = 2\pi f L$$

where f is the frequency of the alternating current in hertz (Hz), and L is the inductance of the circuit or indcutor in henrys (H). Compute the inductive reactance when $f = 10{,}000{,}000$ Hz and $L = 0.015$ H.

9. The minimum retention time (in days) of a certain waste-handling system is given by the expression below. Evaluate the given expression.

$$\frac{1}{0.35\left[1-\sqrt{\dfrac{8100}{8100+121000}}\right]-0.045}$$

10. The impedance in an RC circuit is given by the expression

$$Z_{RC}=\sqrt{R^2+((2\pi f C)^{-1})^2}$$

Determine the impedance if $R = 40$Ω, $f = 60$ Hz, and $C = 8 \times 10$[illegible] F.

Name ______________________________ Date ________________

Exponents and Roots Posttest

Evaluate the following expressions.

1. $(2^3)(2^2)$ Answer ________________

2. $\frac{7^5}{7^2}$ Answer ________________

3. 2^{-3} Answer ________________

4. 9^0 Answer ________________

5. $(3 \cdot 4)^2$ Answer ________________

6. $(3^{-2})^{-3}$ Answer ________________

7. $(4/5)^2$ Answer ________________

8. $\sqrt{4^3}$ Answer ________________

9. $\sqrt[3]{27} * \sqrt[3]{8}$ Answer ________________

10. $(\sqrt[3]{3000}) / (\sqrt[3]{3})$ Answer ________________

11. $\sqrt[3]{\sqrt{729}}$ Answer ________________

12. $(\sqrt[4]{5})^4$ Answer ________________

Instructions for using meter sticks and slide rules:

METER STICK ADDITION AND SUBTRACTION

For all procedures, locate one stick/scale directly **above and adjacent** to the other stick/scale so that both sticks/scales are parallel and fully visible.

For **addition**, locate the **first number** on the **lower stick** and align the **zero** of the **upper stick** with the **first number** still located on the lower stick. Then scan across the **upper stick** to find the **second number**. The **sum** of the two numbers is located **on the lower stick** just below the location of the **second number** on the **upper stick**. (Think of this as finding the sum of the lengths of two strings by placing them end to end to produce the total length.)

For **subtraction**, locate the **first number** on the **lower stick** and align the **second number** on the **upper stick** with the **first number** still located on the lower stick. Then scan across the **upper stick** to find the **zero** of the upper stick. The **difference** of the two numbers is located **on the lower stick** just below the location of the **zero** of the **upper stick**. (Think of this as finding the difference of the lengths of two strings by placing them side by side with one pair of ends aligned so that the difference in lengths is the excess length of the longer string extending beyond the length of the shorter string.)

Hint: To avoid problems with interpreting negative values, use problems such as 7-3 instead of 3-7.

SLIDE RULE MULTIPLICATION AND DIVISION

For multiplication, locate the **first number** on the **lower** D-scale and align the **zero** of the **upper** C-scale with the **first number** still located on the lower D-scale. Then scan across the **upper** C-scale to find the **second number**. The product of the two numbers is located **on the lower** D-scale just below the location of the **second number** on the **upper** C-scale.

For division, locate the **first number** on the **lower** D-scale and align the **second number** on the **upper** C-scale with the **first number** still located on the lower D-scale. Then scan across the **upper** C-scale to find the **zero** of the upper stick. The quotient of the two numbers is located **on the lower D-scale** just below the location of the **zero** of the **upper** C-scale.

NOTICE THAT THE METER STICK AND SLIDE RULE PROCEDURES ARE ESSENTIALLY IDENTICAL; ONLY THE NAMES HAVE BEEN CHANGED WHERE UNDERLINED!

Equipment / materials recommended:

TI-85 Calculators (one per student/person in class)

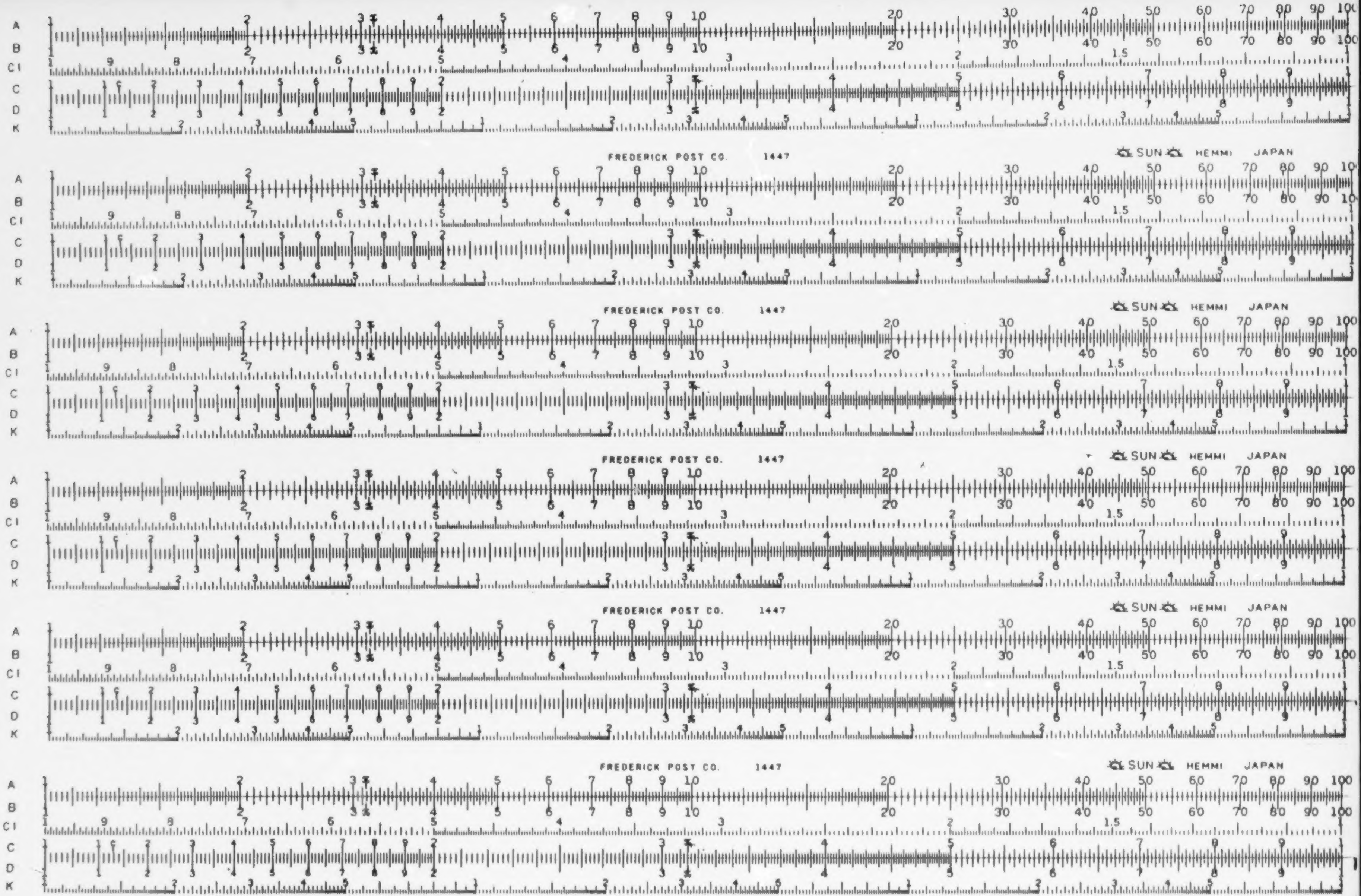
Meter sticks or rulers (two per student)

Slide rules or Slide Rule Simulators (one per student)

Handouts

Recommended textbook: Technical Mathematics from Delmar Publishers

Recommended software: Maple V


A
B
CI
C
D
K
FREDERICK POST CO. 1447
SUN HEMMI JAPAN

COPY TO **CARDSTOCK PAPER**. CAREFULLY **CUT** EACH SEPARATED SLIDE RULE **BETWEEN** THE "C" AND "D" SCALES.

cut

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ESTIMATION SKILLS
APPLIED ALGEBRA CURRICULUM MODULE

***Performance Objective*: Students will be able to perform rounding of numbers and make approximations from world-of-work situations and will be able to make accurate estimations and check for reasonableness of results using interpolation and extrapolation techniques.**

Rounding

Investigation/Demonstration:
The time that it takes for a planet to travel one complete revolution around the sun is called its *period of rotation*. Astronomers and space scientists find this information very useful. The table below give the period of rotation (in years) for each planet.

Planet	Period of Rotation (in years)
Jupiter	11.862
Mars	1.881
Mercury	0.241
Neptune	164.789
Pluto	247.701
Saturn	29.458
Uranus	84.013
Venus	0.616

When we say that a planet takes a period rotation of 247.8 to travel around the sun, we are using a number rounded to the nearest tenth (that is, to *one decimal place*).

The period rotation for the planet Neptune is 164.789
rounded to
the nearest hundredth is 164.79
the nearest tenth is 164.8
the nearest one is 165
the nearest ten is 170
the nearest hundred is 200

Complete these:
The period rotation for the planet Pluto is 247.701
rounded to
the nearest hundredth is
the nearest tenth is
the nearest one is
the nearest ten is
the nearest hundred is

NOTE: Many industry applications use a special rule for rounding when the position of the number up for consideration for rounding is a "5". The generally accepted rule in this situation: The number to the left of the number '5' is rounded to the nearest *even* number.

Using the example for the planets, let us consider this example:

The period rotation for the planet Saturn	is 29.458
rounded to	
the nearest hundredth	is 29.46
the nearest tenth	is 29.5
the nearest one	is 30

The period rotation for the planet Pluto	is 247.701
rounded to	
the nearest hundredth	is 247.70
the nearest tenth	is 247.7
the nearest one	is 248
the nearest ten	is 250
the nearest hundred	is 200

Activity

Have you ever made a long-distance call? Long-distance charges are based upon

- where you make the call to
- what time of day that you make the call, and
- how long your call lasts

A discount is given for some calls. The table below shows a schedule of discount rates for long-distance calls between cities in the United States.

Schedule of Discount Rates

Monday to Friday	8:01 am to 6:00 pm	No discount
Monday to Friday	6:01 pm to 11:00 pm	1/3 discount
Saturday	8:01 am to 12:00 noon	No discount
Saturday	12:01 pm to 11:00 pm	2/3 off
Sunday	8:01 am to 6:00 pm	2/3 off
Sunday	6:01 am to 11:00 pm	1/2 off
Daily	11:01 pm to 8:00 am	2/3 off

The long-distance charge on a call made by Jeff from Dallas, Texas to Nashville, Tennessee on Wednesday at 10:00 pm for 5 minutes is calculated at $0.53 per minute as:

Long-distance charge = (Regular rate per minute X minutes) - Discount
= ($0.53 X 5) - 1/3 Discount
= $2.65 - 1/3 X $2.65 = $2.65 - $0.88 = $1.77

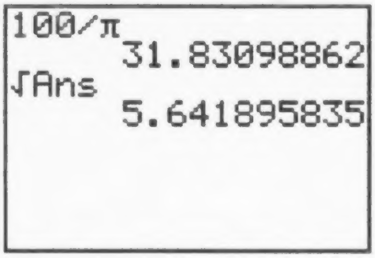
- Doug called his friend long-distance on Saturday at 11:00 am. He talked for 6 minutes. What did this long-distance call cost if the regular rate was $0.67 per minute?
- Jovani made a long-distance call from Phoenix, Arizona to Cheyenne, Wyoming on Sunday at 9:00 am. He talked for 12 minutes. What did his call cost if the regular rate was $0.58 per minute?

Approximation

Investigation/Demonstration:

A piston in a small engine is to be designed such that the surface area of the top of the piston is to be as close to 100 square centimeters as possible. What should be the measure of the radius of the piston to the nearest 0.1 centimeter that will give us a surface area as close to 100 square centimeters?

Recall that the area of the circle is ______ so 100 = ______. Solving for the radius r we get the calculated result seen on the calculator display below:



```
100/π
          31.83098862
√Ans
          5.641895835
```

Approximately how much error in the surface area will result due to rounding the radius to 5.6 centimeters?

Approximately how much error in the surface area will result due to rounding the radius to 5.7 centimeters?

Activity

The distance along interstate highway 30 from Texarkana to Dallas is 180 miles. Traveling west from Texarkana (milepost 0) to Dallas (milepost 180) you will pass near the cities of Mount Pleasant (milepost 58), Mount Vernon (milepost 80), Sulphur Springs (milepost 100), Greenville (milepost 135), and Rockwall (milepost 150). If Joey is traveling at a constant rate of 65 miles per hour along the route from Texarkana to Dallas, approximate the number of minutes that it takes to travel from:

- Texarkana to Mount Pleasant
- Mount Pleasant to Mount Vernon
- Mount Vernon to Sulphur Springs
- Sulphur Springs to Greenville
- Greenville to Rockwall
- Rockwall to Dallas
- Texarkana to Dallas

Extrapolation

Investigation/Demonstration:

Graphs and tables can be used to observe patterns and obtain information concerning a relation in order to make a prediction. The use of data in finding exact answers or in making predictions in this way is called *extrapolation*. Linear data patterns lend themselves well to such predictions. Suppose a pizza parlor is considering making a larger size pizza, 24 inches in diameter. Presently the three sizes they presently sell are 6 inch, 9 inch, and 12 inch diameter pizzas. The price of the pizza depends upon the diameter of the pizza.

DIAMETER	PRICE
6 inch	$3.95
9 inch	$4.95
15 inch	$6.95
24 inch	??

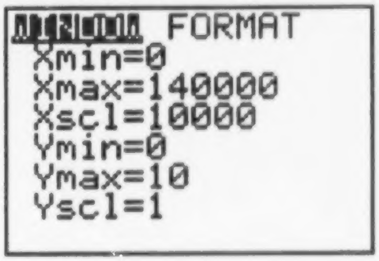
What should be the price for the 24-inch diameter pizza?

How did you arrive at your answer?

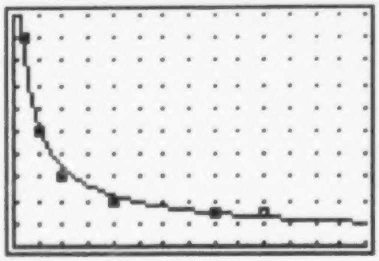
Activity

Use the technique of extrapolation to obtain information concerning a relation that is non-linear. The data below show the cost to a company for manufacturing various quantities of CD's.

The graph shows the relation is non-linear.



```
WINDOW FORMAT
Xmin=0
Xmax=140000
Xscl=10000
Ymin=0
Ymax=10
Yscl=1
```



The dimensions for the viewing window for the graph is [0, 160000] with a scale factor of 10000 and [0, 10] with a scale factor of 1.

Number	Cost
5,000	$9.00
10,000	$5.00
20,000	$3.00
40,000	$2.00
80,000	$1.50
100,000	$1.40

- Estimate the cost of manufacturing 120,000 CD's.
- Estimate the cost of manufacturing 150,000 CD's.
- Estimate the number of CD's to be manufactured if the cost is to be $1.30.
- Estimate the number of CD's to be manufactured if the cost is to be $1.25.

Interpolation

Investigation/Demonstration:
Graphs and tables can be used to observe patterns and obtain values between given data. The use of data in finding exact answers or in making predictions within pairs of data in this way is called *interpolation*. Linear data patterns lend themselves well to such predictions. Suppose the pizza parlor in the example above is considering making a pizza that is 12 inches in diameter. Presently the three sizes they presently sell are 6 inch, 9 inch, and 12 inch diameter pizzas. The price of the pizza depends upon the diameter of the pizza.

DIAMETER	PRICE
6 inch	$3.95
9 inch	$4.95
15 inch	$6.95

What should be the price for the 12-inch diameter pizza?

How did you arrive at your answer?

Activity

Use the technique of interpolation to obtain information concerning a relation that is non-linear. Refer again to the table below which shows the cost to a company for manufacturing various quantities of CD's. The graph shows the relation is non-linear. The dimensions for the viewing window for the graph is [0, 160000] with a scale factor of 10000 and [0, 10] with a scale factor of 1.

Number	Cost
5,000	$9.00
10,000	$5.00
20,000	$3.00
40,000	$2.00
80,000	$1.50
100,000	$1.40

- Estimate the cost of manufacturing 50,000 CD's.
- Estimate the cost of manufacturing 75,000 CD's.
- Estimate the number of CD's to be manufactured if the cost is to be $2.25.
- Estimate the number of CD's to be manufactured if the cost is to be $4.00.

Reasonableness of Results

Investigation/Demonstration:

To solve problems, you often need to do calculations with decimal numbers. When your work is completed, you should always ask, as a check "is my answer reasonable?" This skill is very important when using a calculator. Calipers and micrometers are used to find precise answers. Estimation skills are important in determining reasonableness of results.

Activity

- Estimate the cost of 21 concert tickets at $8.95 each. Will two one-hundred dollar bills be enough to pay for the 21 tickets? Explain your answers.

- Five tubes of a specialized paint cost a total of $19.43, including tax. What is the cost of twenty tubes of the paint?

- The cost for installing a new roof is $48.75 per square yard. Estimate the cost of roofing a shed that has 97 square yards, a garage that has 318 square yards, and a warehouse that has 594 square yards.

- With a caliper, measure the following to the nearest millimeter--the width of your eraser, the length of your index finger, and the width of your wrist.

Evaluation Instrument

1. Estimate the cost of a new car that has a base list price of $11,799 with the following additional items as options:

CD player	$795	Sun roof	$699	TV	$185

2. On the day that Clarice exchanged American dollars for German marks she found that each dollar would buy 2.28 marks. How many marks did she receive for $250?

For questions 3-5, use this information: A building contractor estimates that he will need the following materials to completely re-do a bathroom: 5 sheets of drywall at $8.59 per sheet; 10 pounds of compound at $5.85 per 5 LB can; 1 roll of tape at $2.18; 3/4 pounds of nails at $0.98 per pound; 1 can of sealer at $18.95; 1 can of paint at $16.95; and 4.5 yards of baseboard at $10.29 per yard.

3. Find the estimated cost for materials

4. The contractor charges $45.00 per hour for labor and he estimates that the job will take 25 hours to complete. Find the estimated cost for labor.

5. What is the total estimated cost to re-do the bathroom?

Materials required for teaching this unit:

TI-82 Graphing Calculators
TI-82 Viewscreen
Grid Paper
Measuring Tapes
Micrometers
Calipers

Word Problems

APPLIED ALGEBRA CURRICULUM MODULE

PERFORMANCE OBJECTIVE:

Upon completion of this module the learner will demonstrate at the excellent, good, average, or no credit performance level the oral, written, teamwork, and calculation mastery of the concepts and procedures most often identified in the development of effective skills for solving word problems.

STATEMENT OF CONNECTION:

No area of mathematics causes students any more difficulty than word problems. There are several reasons that explain this situation. Many intermediate and college algebra students simply do not comprehend what they have read or heard. The technical readability level of the material is often higher than their performance level. In having students read instructions from their textbook exercise sets, I have found that many are poor readers who stumble over words like simplify and expression. When they complete reading the instructions, I sometimes inquire did anyone understand that. Only a few respond that they definitely understand what they were instructed to do. This reminds me of a cartoon I once had clipped on the bulletin board outside my office. The teacher says, "The distance that an object falls is directly proportional to the square of the time it falls." The students hear, "Shing Boh Han Shun Ning Ka La."

Since becoming a better reader and listener is prerequisite to becoming a better word problem solver, patience and perseverance must be encouraged. Gross improvement will not occur overnight. The following activity allows the learner to actively participate as an individual and team member in the process of becoming a better word problem solver.

ACTIVITY: RELATIONSHIPS IN ELECTRICITY:

The idea for this activity came from the fact that the study of electric circuits is fundamental to most technical training programs and is certainly a hands on part of many work places. Students will research, read, write, discuss, and calculate with electricity.

PROCEDURES:

Divide the class into groups of two to four and instruct them to have a small group discussion regarding electricity. Remind them that their oral responses are being graded. Before dismissing, ask them to work alone and briefly research the topic of electricity providing a paragraph to be submitted for a grade. At the beginning of the next class meeting ask them to share their paragraphs in small groups then have an entire class share session. Some of the paragraphs may deal with video games, home appliance, lightning, static electricity, or even Ohm's law. To help focus this discussion provide **Important Contributions in Electricity**, Appendix A. Explain to the class that the key ingredients in becoming a better word problem solver are reading comprehension and translation. For example in English using words we say the area of a rectangle is equal to its length times its width.

In mathematics using symbols we say the same thing,

$$A_{rec} = L \cdot W$$

Symbols:

A - area of a rectangle

rec - abbreviated subscript referring to rectangle

= - equals

L - length

· - times or multiplication

W - width

Stress to the class that both they and you must become knowledgeable with regard to their technical reading ability. Administer the diagnostic examination, Appendix B, for a grade. Tell them to read carefully Appendix A before the next class and write on their own paper to be submitted for a grade the sections dealing with volts, amps, and ohms. Also, ask them to include the three important quantities related in Ohm's law with their own symbols for each and an equation that might relate them. (Remember $A_{rec} = L \cdot W$). At the beginning of the next class allow them time to share in small groups. Some lecture time will be needed to discuss the actual relationship and standard symbols for current, voltage, and resistance. After this, in groups of two, they should have a laboratory experience on a DC circuit board with a variety of resistors and variable power supply. It should be designed so that students using a digital multimeter can measure and record resistance, current, and voltage at various points in the circuit. For safety purposes the instructor should hand out Appendix C one day and ask the class to submit a list of nine complete sentences the following day in which they have incorporated one good why for each of the nine rules. After a class discussion on safety, the instructor should collect the lists for a grade. The safest and surest laboratory design is for the instructor to set up and monitor **ONE** DC circuit board where teams of two come and are instructed in the use of the digital multimeter. The circuit should allow each team the opportunity to measure and record at least three different relationships of I, V, and R. Stress the importance of accurate records, sketches of the circuits, and appropriate units of measurement. Typical examples are shown in Appendix D. After all teams have collected and recorded their data, ask them to verify that the results satisfy the conditions of Ohm's law, $I = V/R$ and alternately, $V = IR$. Also, ask them to graph one of their scenarios of collected data possibly voltage on the horizontal, current on the vertical, and a constant resistance (Appendix D). The laboratory data organization, sketches of circuits, formula verifications, and graph will all be collected and graded. In the next class meeting some lecture time will help bring closure to the study of word problems. The following plan may be shared at this time with the disclaimer, "This plan and your hard work will help you become a better word problem solver".

"THE PLAN"

1. Read or listen to the entire problem **UNTIL** you somewhat **UNDERSTAND** the situation.
2. Research any unfamiliar terms and record pertinent recalled relationships.
3. Read or listen again to the problem in pieces listing the knowns in a table and sketching a picture if possible.
4. Recall or research the appropriate formulas or relationships.
5. Read or listen again for the unknown and decide what variable to use.
6. Translate the work thus far into a formula, equation, inequality, etc., involving the relationship of knowns and unknown.
7. Solve the equation, inequality, etc.
8. Check all solutions.
9. Read or listen one last time to be certain that everything has been resolved correctly.

EVALUATION INSTRUMENTS:

1.	Paragraph	Research the Topic Electricity
2.	Reading Analysis	Appendix A
3.	Oral	Small group and large group discussions
4.	Diagnostic Test	Appendix B
5.	Laboratory	Data collections, circuit sketches, formula verifications, graph
6.	List of Whys	Appendix C
7.	Performance Examination	Scramble or revise the Diagnostic test adding parts from "The Plan"
8.	Essay	What did you learn? Be thorough! Have you usually had trouble with word problems? Will you feel more confident in the future?

SUPPLIES:

1. Pencils and Paper
2. Calculators
3. Digital Multimeter
4. Variable Power Supply
5. DC Circuit Board
6. Variety of Resistors

APPENDIX A
IMPORTANT CONTRIBUTIONS IN ELECTRICITY

William Gilbert (1540 - 1603), an English physician, described how amber differs from magnetic loadstones in its attraction of certain materials. He found that when amber was rubbed with a cloth, it attracted only lightweight objects, whereas loadstones attracted only iron. Gilbert used the Latin word *elektron* for amber and originated the word *electrica* for the other substances that acted similarly to amber.

Sir Thomas Brown (1605 - 1682), an English physician, is credited with first using the word electricity.

Stephen Gray (1696 - 1736), discovered that some substances conduct electricity and some do not.

Charles du Fay experimented with the conduction of electricity. These experiments led him to believe that there were two kinds of electricity. He found that objects having vitreous electricity repelled each other and those having resinous electricity attracted each other.

Benjamin Franklin (1706 - 1790) conducted studies in electricity and was the first to use the terms *positive* and *negative*. In his famous kite experiment, Franklin showed that lightning is electricity.

Charles Augustin de Coulomb (1736 - 1806), a French physicist, proposed the flaws that govern the attraction and repulsion between electrically charged bodies. Today, the unit of electrical charge is called the *coulomb*.

Alessandro Volta (1745 - 1827), an Italian professor of physics, discovered that the chemical action between moisture and two different metals produced electricity. Volta constructed the first battery, using copper and zinc plates separated by paper that had been moistened with a salt solution. This battery, called the *voltaic pile*, was the first source of steady electric current. Today, the unit of electrical potential energy is called the *volt*.

Hans Christian Oersted (1777 - 1851), is credited with the discovery of electromagnetism. He found that electrical current flowing through a wire caused the needle of a compass to move. This finding showed that a magnetic field exists around a current-carrying conductor and that the field is produced by the current.

Andre' Ampere (1775 - 1836), a French physicist, measured the magnetic effect of an electrical current. He found that two wires carrying current can attract and repel each other, just as magnets can. By 1822, Ampere had developed the fundamental laws that are basic to the study of electricity. The modern unit of electrical current is the *ampere* (also called *amp*).

Georg Simon Ohm (1787 - 1854), a German teacher, formulated one of the most noted and applied laws in electrical circuits, Ohm's law. Ohm's law gives the relationship among the three important electrical quantities of *resistance, voltage,* and *current*.

APPENDIX B
DIAGNOSTIC TEST

I. Choose one of A, B, C, D or E listed below as being the best related to the word given.

A) + C) x E) =
B) - D) ÷

____ 1. Multiply
____ 2. Difference
____ 3. Divide
____ 4. Sum
____ 5. Equals
____ 6. Product
____ 7. Plus
____ 8. Subtract
____ 9. Times
____ 10. Is
____ 11. Add
____ 12. Minus
____ 13. Quotient

II. Choose one of A, B, C, D or E listed below as being the best related to the phrase given.

A) 6 + 2 C) 6 x 2 E) 2 - 6
B) 6 - 2 D) 6 ÷ 2

____ 14. Six times two
____ 15. Six plus two
____ 16. Six divided by two
____ 17. The product of six and two
____ 18. Two subtracted from six
____ 19. Quotient of six and two
____ 20. The sum of six and two
____ 21. Two less six
____ 22. Two less than six

III. Choose one of A, B, C, D or E listed below as being the best related to the sentence given.

A) $2 + 3 = 5$ C) $5 - 2 = 3$ E) $6 \div 2 = 3$
B) $2 \times 3 = 6$ D) $6 \div 3 = 2$

____ 23. Two times three is six.
____ 24. The quotient of six and three is two.
____ 25. Two less than five is three.
____ 26. The sum of two and three is five.
____ 27. Three is two less than five.
____ 28. Twice three is six.
____ 29. Six divided by two is three.
____ 30. Five minus two is three.
____ 31. Two plus three is five.
____ 32. The product of two and three is six.
____ 33. Two more than three is five.
____ 34. Two subtracted from five is three.
____ 35. The difference between five and two is three.
____ 36. Two added to three is five.
____ 37. Five exceeds two by three.
____ 38. The result of three multiplied by two is six.
____ 39. Six divided by three is two.
____ 40. The result of dividing two into six is three.

IV. Choose one of A, B, C, D or E to indicate how the problem should be solved.

____ 41. Randy rides his bike 14 miles every morning to deliver papers. If he delivers papers each day of the week, how many miles does he ride in a week?

A) $7 + 14$ B) $14 - 7$ C) 7×14

D) $14 \div 7$ E) $7 \div 14$

____ 42. Lori had a birthday last week. She got $18 from her aunt and $14 from her grandmother. How much money did she get in all?

A) $14 + 18$ B) $18 - 14$ C) 14×18

D) $18 \div 14$ E) $14 \div 18$

43. Bill is going on a two-day trip. He will have to buy three meals each day. He saved $24 for the trip. If he spends all of this on meals, how much does he spend each day on meals?

A) 24 + 2 B) 24 ÷ 6 C) 24 x 2

D) 24 ÷ 3 E) 24 ÷ 2

____ 44. Sue wants to buy a ring. The ring costs $99 and she has saved $66 toward the purchase of the ring. How much more does Sue need in order to buy the ring?

A) 99 + 66 B) 99 - 66 C) 99 x 66

D) 99 ÷ 66 E) 66 - 99

____ 45. Mark got his check of $196 for the week. If he bought a CD for $13, how much does he have left.

A) 196 + 13 B) 196 - 13 C) 196 x 13

D) 196 ÷ 13 E) 13 - 196

____ 46. Cindi has two jobs. She earns $5 an hour on one of them and $8 an hour on the other. How much does Cindi earn each hour she works at both jobs?

A) 8 + 5 B) 8 - 5 C) 8 x 5

D) 8 ÷ 5 E) 5 ÷ 8

____ 47. Larry works at a part-time job after school. He makes $26 each night. If he works three hours each night, how much does he make an hour?

A) 26 + 3 B) 26 - 3 C) 26 x 3

D) 26 ÷ 3 E) 3 ÷ 26

For the following three problems use the formula,

$$I=\frac{V}{R}$$

____ 48. Find V if I = 0.5 and R = 10.

A. 20 B. 0.05 C. 5 D. 10.5 E. 9.5

____ 49. What is the value of R when I = 2 and V = 5?

A. 10 B. 0.4 C. 3 D. 7 E. 2.5

____ 50. Calculate I for R = 5 and V = 20.

A. 0.25 B. 4 C. 25 C. 100 E. 15

APPENDIX C
LABORATORY SAFETY RULES

1. Never work alone.
2. Wear appropriate clothing and safety equipment.
3. Remove all jewelry.
4. Keep an organized work station.
5. Know the location of emergency switches, equipment, and supplies BEFORE any laboratory work transpires.
6. Ensure that all equipment is in good working condition BEFORE using it.
7. Use instruments only as they were designed.
8. If uncertain about a procedure, ask the instructor BEFORE performing it.
9. Report any unsafe or questionable condition to the instructor.

Problem Solving - Rates

APPLIED ALGEBRA CURRICULUM MODULE

PERFORMANCE OBJECTIVE:

Upon completion of this module the learner will demonstrate at the excellent, good, average, or no credit performance level the oral, written, teamwork, and calculation mastery of the concepts and procedures of problem solving involving rates.

STATEMENT OF CONNECTION:

The basic objective of any language is the communication of ideas. It is critically important to recognize that mathematics is the language of technical problem solving. Advantages associated with mathematical development are very real and can be used to enhance problem solving skills in a variety of arenas including work, play, interpersonal relations, as well as college training. Problem solving, regardless of the problem nature, begins with an effort to clearly define the problem. A problem that is **NOT UNDERSTOOD** neither can be efficiently nor effectively resolved. One activity, though not totally workplace related, allows the learner to actively participate as an individual and team member in problem solving from the vague inception to personally meaningful results.

ACTIVITY: MATHEMATICS CAN SAVE YOUR LIFE

The idea for this activity came from an old edition of

College Algebra by Fleming and Varberg in which a cartoon was presented called "Johnny's Dilemma".

> Although he had been warned against it a thousand times, Johnny still walked across the railroad bridge when he was in a hurry which was most of the time. Today he is one-fourth of the way across the bridge and he notices a train coming one bridge length away. Which way should he run?

PROCEDURES:

Divide the class into groups of two to four and give each member a copy of "Johnny's Dilemma". Explain to them that many problems from real life are similar and before they can resolve "Johnny's Dilemma" they must **UNDERSTAND** the problem. Allow them a few minutes for small group discussion then ask them to work alone until the next class when they will submit a handwritten paragraph responding to the question: Do you think Johnny was really warned a thousand times? Emphasize to them the need for good written communication skills. Also, tell them to compile two lists of questions. The first list will be primary which contains the critical knowns like how fast is the train coming, etc. The second list will be secondary which includes avenues of escape like can Johnny swim, etc. These lists will be shared within small groups then shared to entire group and a fantastic discussion should ensue. These lists should be collected and graded. In the discussion it should be confirmed that "which way should he run" means to save his life, because if he had indeed been warned a thousand times perhaps Johnny had a death wish.

Explain to the class that their oral participation in small group and large group is being evaluated. Before class is over give them some of the more important knowns such as the train's speed is fifty miles per hour and the bridge is forty meters long. Ask them for the next class meeting to prepare a scaled drawing of the situation. This also will be collected and graded. Notice the units are English and Metric which will allow for conversions and dimensional analysis later on in the calculations section of this activity. The next class meeting will be data collection time. Each group will need measuring devices for distance (perhaps forty meters) and time (stop-watches). Allow each group to simulate Johnny and the bridge and calculate his speed in getting off the bridge in meters per second. Tell them to collect all the data they think they will need to resolve the problem and record their data in a very meaningful manner for presentation. Be sure to stress the level of precision, accuracy, and significant digits required. In the next class meeting some lecture time will help firm up the meaning and relationship among distance, rate, time, units, conversions, velocity, speed, acceleration, de-acceleration, dimensional analysis, pertinent formulas, and use of the scientific calculators.

EVALUATION INSTRUMENTS:

1. Paragraph "Do You Think Johnny Was Really Warned a Thousand Times"

2.	Lists	Primary and secondary questions
3.	Oral	Small group and large group discussions
4.	Drawing	Scale of train, bridge, and Johnny
5.	Data	Report of data on "Johnny's speed"
6.	Calculations	In class use 50 mph train speed and the lab measured "Johnny speed" and convert both to feet per second. Show a sketch of the point of impact emphasizing several scenarios toward train, away from train, train from behind, train from front, etc.
7.	Essay	What did you learn? Be thorough. Have you ever tried to beat a train at an intersection? Will you in the future?

SUPPLIES:

1. Pencils and paper
2. Rulers Metric/English
3. Calculators
4. Tape measures Metric/English
5. Stop Watches

Linear Equations and Inequalities

APPLIED ALGEBRA CURRICULUM MODULE

Objectives

Section 1: After completing this section, you will be able to:

- graph an equation in two variables on the graphing calculator
- find the complete graph of an equation
- use the ROOT and ZOOM functions to find the x-intercepts and the y-intercepts of a graph as accurately as required by a particular problem
- interpret the x-intercepts of a graph in an application problem

Section 2: After completing this section, you will be able to :

- determine if an equation represents a line
- determine the slope of the line both from the graph and by using a formula
- determine the equation of a line given a point and slope
- determine the equation of a line given two points
- interpret the slope of a graph with respect to an application problem

Section 3: After completing this section, you will be able to:

- solve systems of equations in 2 variables on the calculator using ISECT and ZOOM
- set up and solve systems of equations in application problems

Section 4: After completing this section, you will be able to:

- use interval notation to describe the range of a variable
- solve inequalities in one and two variables using the graphing calculator
- determine the boundaries of a system of inequalities
- determine the corner points of a system of inequalities

Section 5: After completing this section, you will be able to:

- use inequalities to solve maximum and minimum problems involving constraints.

Section 1

Hospitals carry many solutions of medications in their pharmacies. However, they do not carry every strength necessary. So, mixing a solution is an important aspect of being a pharmacist.

Mark has been asked to prepare 500 ml of a 15% dextrose intravenous solution for a premature baby. He has a 70% dextrose solution on hand that he can mix with pure water. How much of each solution does he need to use in order to make the required solution?

In order to solve this problem, we will need to

1. decide what the unknown quantities are,
2. establish relationships between the unknown quantities and the known quantities,
3. set up an equation, and
4. solve the equation and interpret the solution.

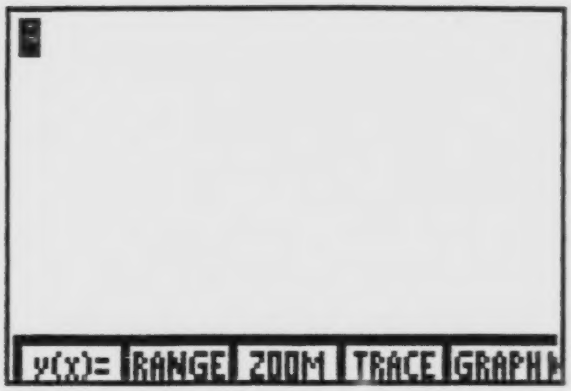
Suppose for a moment that we have reached step 3, namely, that we have set up the equation already. How do we solve it? We will solve the equation on the calculator by graphing it. Before we go back to finishing the pharmacy problem, we must first learn how to graph an equation on the calculator and how that graph will help us solve the problem.

Note: All of the keystrokes shown in the following examples will be for the TI-85. If you have a different type of graphing

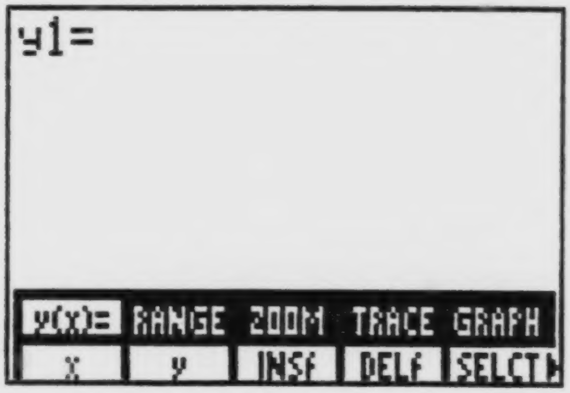
utility, use the instruction manual for your machine to determine how to do the steps.

Example 1 Graph y = 3x + 12

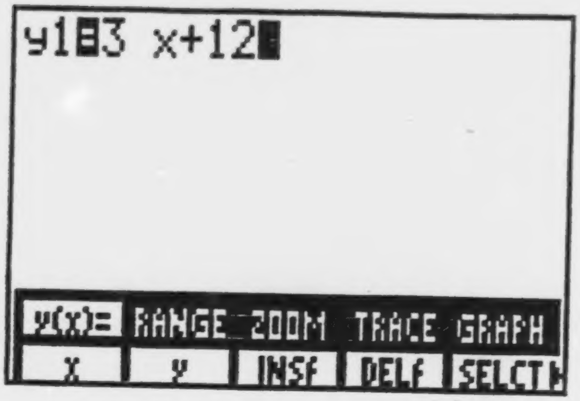
Solution: Turn on the calculator and push CLEAR twice. This screen is called the home screen. Now press GRAPH. Your screen should look like:


y(x)=
RANGE
ZOOM
TRACE
GRAPH

Press F1 (y(x)). This will give the screen:


y1=
y(x)=
RANGE
ZOOM
TRACE
GRAPH
x
y
INSf
DELf
SELCT

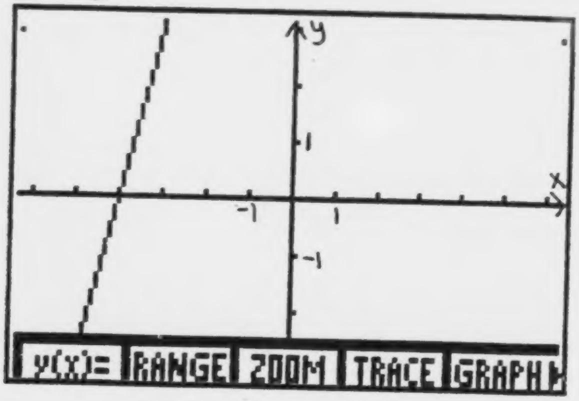
Type in 3x + 12: 3 XVAR + 12.



y1=3 x+12

y(x)=
RANGE
ZOOM
TRACE
GRAPH
x
y
INSf
DELf
SELCT

Press 2nd F3 (zoom), then press MORE. At F4 you will see ZDECM. This will make the scale on the screen have spacing like on graph paper. This is what is called a "friendly" window. press F4. This makes the graph appear:



y
x
1
-1
1
-1
y(x)=
RANGE
ZOOM
TRACE
GRAPH

This view of the graph shows where the graph of the equation crosses the x-axis, or the horizontal axis. Press F4 (Trace). You will notice that at the bottom of the screen you have:

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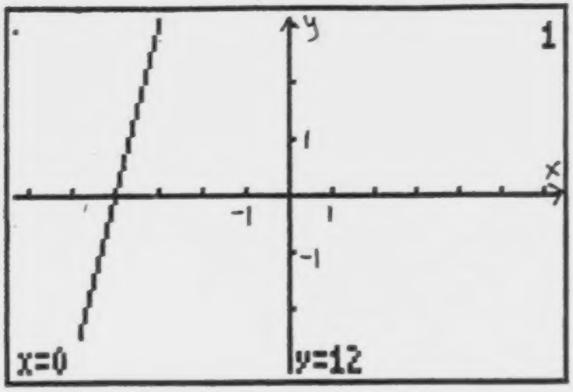

Press the left cursor a few times and then press the right cursor for awhile. What is happening? The graph adjusts itself to show where the cursor is when it seems like it has moved off the screen. This means that all we are seeing when we first graph the equation is just a portion of the graph.

Look at the graph for a few minutes. Use the cursors to move to the left and the right. Are you able to draw a conclusion about the shape of the graph in general? In other words, are you able to give a description of the graph that would seem to include all of the features of it? (Sometimes you must piece together to windows if the graph is really strange.) If you are able to do this, then you have a **complete graph**. Our original graph was not a complete graph because when we first saw it, it did not cross the y-axis. After we had traced for awhile, we found that it did cross the y-axis. This means that our **window** was not as good as it could have been.

In general we want to be able to see the x-intercepts (where

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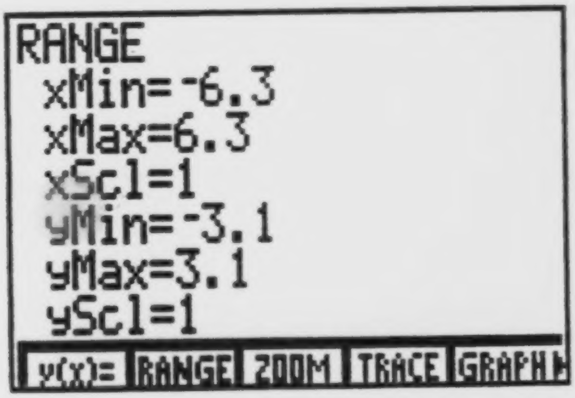
y = 0), the y-intercepts (where x = 0), and any hills and valleys, if the window that allows this is reasonable and doesn't cause the shape of the graph to become too distorted.

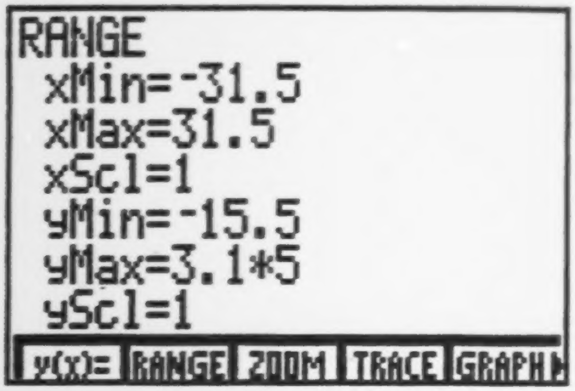
Let's examine the window that we are using. The calculator graphs equations by picking values for x and then finding out what y is. Then it plots those points and connects them to form the graph. There are 127 points in the horizontal direction and 93 points in the vertical direction. Thus the distance across the screen is 126 and the distance vertically is 92. As long as we keep the window at a multiple of these numbers, we have what is called a friendly window. Retrieve the Range screen on your calculator. (In GRAPH, press F2, range.)

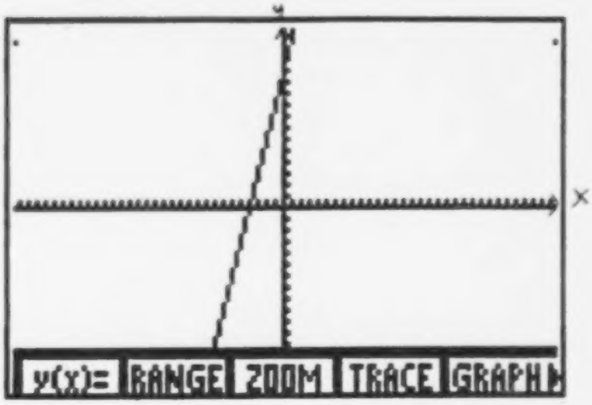


RANGE
xMin=⁻6.3
xMax=6.3
xScl=1
yMin=⁻3.1
yMax=3.1
yScl=1

Right now it has the ZDECM values on it since that was what we used in Example 1. Notice that xmax - xmin = 6.3 - (-6.3) = 12.6. This is a multiple of 126, namely, **0.1**(126). Likewise, ymax - ymin = 3.1 - (-3.1) = 6.2 = **0.1**(6.2). We will sometimes change the values that are in bold above, the 0.1. Both of them do not have to be the same, as you will see in some of the following examples.

Now go back and let's try to get a complete graph of this equation. Go into RANGE and multiply the x values by 5 and the y values by 5. These are arbitrary numbers; you can always go back in if they were too big or not big enough.


RANGE
xMin=-31.5
xMax=31.5
xScl=1
yMin=-15.5
yMax=3.1*5
yScl=1



Now we have a complete graph, because earlier, we notice that the graph would be a line and it crossed the x-axis once and the y-axis once, and now those places are shown.

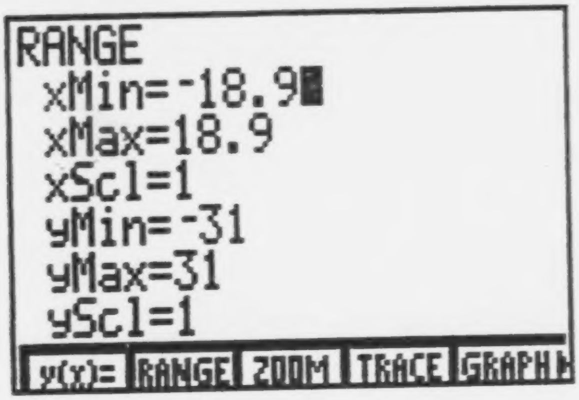
Example 2: Graph $y = -2x - 24$.

Solution: Following the same steps that we did at the beginning of Example 1, graph this equation.

(fig 1.8)

Nothing showed up on the graph, did it? That is because if x is between -6.3 and 6.3, the y-values will not be between -3.1 and 3.1. (Remember that is the window we are using.) This happens a lot of times. So, again, we must adjust our range, or window, so that the graph will appear on the screen. Press F2 (range) and multiply the x and y values to get a larger spread or range for x and a larger range for y. For example:

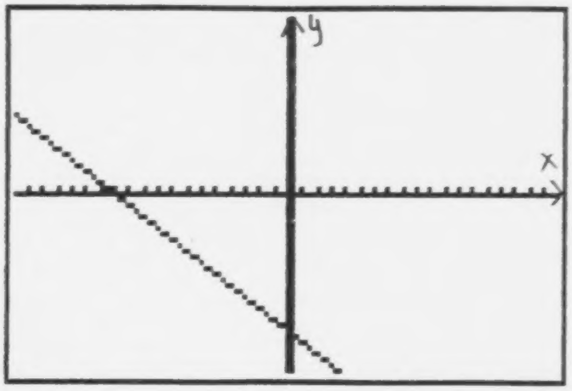
multiply
xmin by 3 xmax by 3
ymin by 10 ymax by 10



```
RANGE
 xMin=-18.9
 xMax=18.9
 xScl=1
 yMin=-31
 yMax=31
 yScl=1
```

and this will give a new friendly window.

Now press F5 to graph.



Notice that even if you chose different values to multiply by, the graph would still cross the x- and y-axes at the same points.

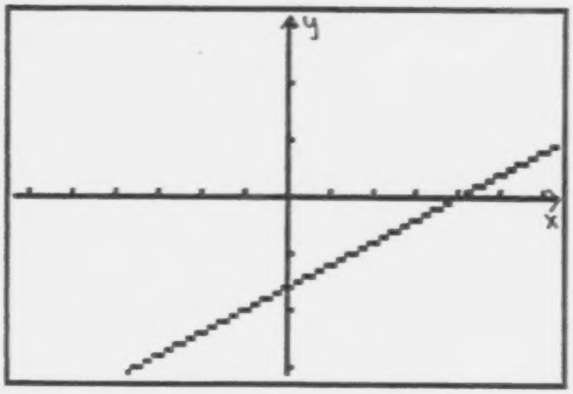
(Try it and check that with TRACE.)

Example 3: Graph $2x - 5y = 8$.

Solution: First, we must solve for y because the calculator only excepts expressions with one variable, x, to graph. Thus we will graph the equation $y = \frac{2}{5}x - \frac{8}{5}$. Be very careful with parentheses when you enter the equation into the calculator:

Enter (2 / 5) xvar - (8 / 5)

The graph in ZDECM is:



How does the graph help us solve an equation? Recall that when we solve an equation, we are looking for the value of the variable that makes the equation true. Consider the equation $3x + 12 = 0$. In example 1, we graphed $y = 3x + 12$. How are these two equations similar? How are they different? If $y = 0$, then both of these equations are saying the same thing. This means that if we look at the graph and try to find the x value for which $y = 0$, then we will have solved the equation.

Example 4: Solve $4x - 5 = 7x + 15$ by graphing.

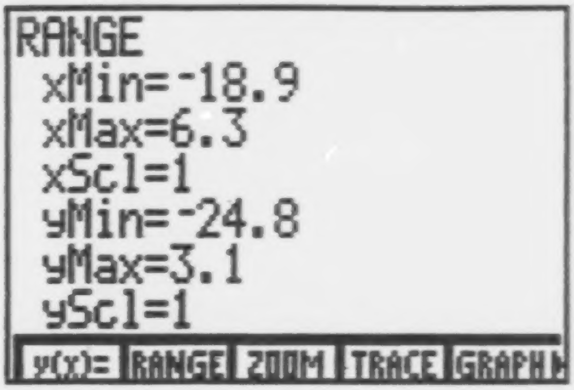
Solution: Get everything on one side first with zero on the other.

$$4x - 5 - 7x - 15 = 0$$

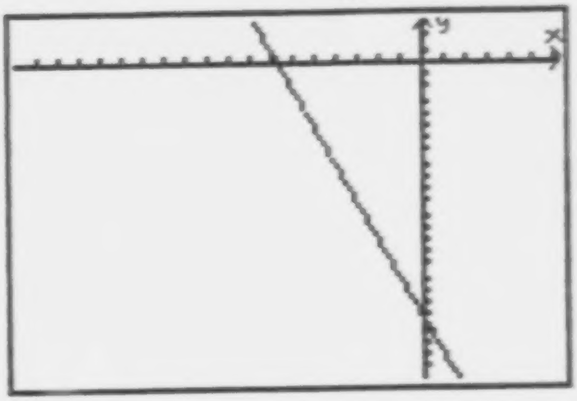
$$-3x - 20 = 0$$

Now, graph $y = -3x - 20$. The line is jagged, but that is the way graphs sometime appear on the calculator.

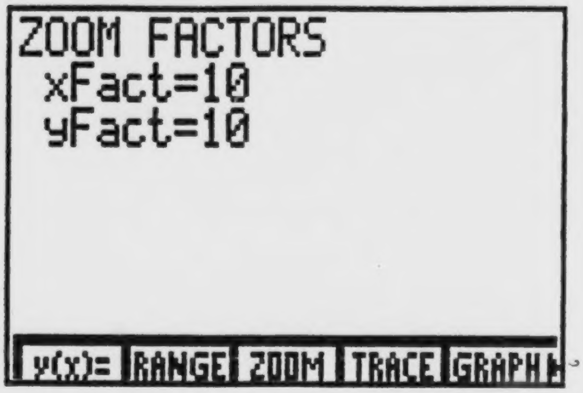
The range used here is: and the graph is:



RANGE
xMin=⁻18.9
xMax=6.3
xScl=1
yMin=⁻24.8
yMax=3.1
yScl=1

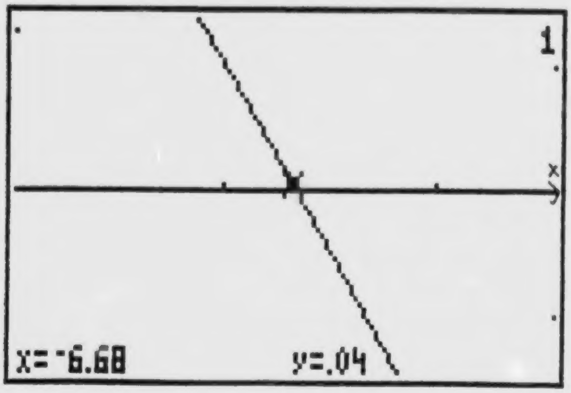


Notice that when we are using the TRACE (F4) to look for where $y = 0$, the y values jump from $y = 0.1$ to $y = -0.5$. We cannot get zero exactly. Let us enlarge this portion of the graph so that we may see those values closer. We will ZOOM in on this place. To get the menu back along the bottom of the screen, push EXIT. Then press F3 (zoom). Press MORE twice. Before we ZOOM, we need to check the zoom factors. These determine the magnification that the calculator will use, just like on a microscope or a telescope. Press F1 (ZFACT).



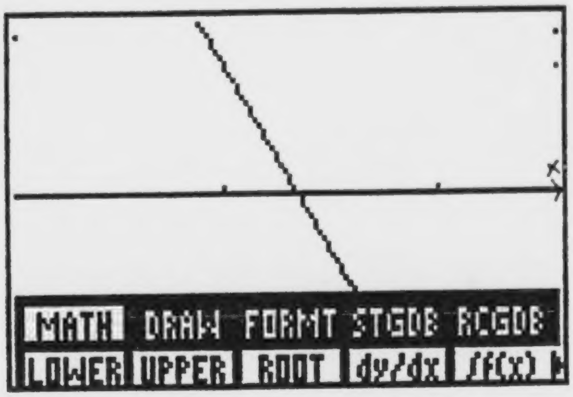
ZOOM FACTORS
xFact=10
yFact=10

Make sure your factors are 10. This will allow our graph scale to remain "friendly" by just moving the decimal point. (The scale will always remain friendly, no matter what your zoom factors are, as long as you started with a friendly window.) Now press F3 (zoom) again. Press F2 (ZIN) This gives the graph with a flashing cursor at (0,0). Move the cursor to the left until it is near the place where the graph intersects the x-axis. Press enter. This gives:

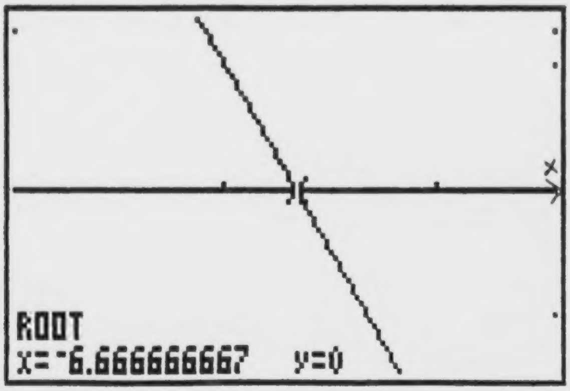


x=⁻6.68 y=.04

Press EXIT to return the menu. Press F4(trace) and with the right cursor, trace to where y = 0. This is called the x-intercept. We have: x = -6.66, y = -0.02 and x = -6.68, y = 0.04. We still are not on the place where y = 0 exactly. Press EXIT then MORE. Press F1(Math). Then press F3 (root).



This will give the x-value of the graph exactly at y = 0. (We could have done this instead of zooming in from the very start.) Move the cursor near the intersection point and press ENTER.



ROOT
x=-6.666666667 y=0

The root is x = -6.666666667, or x = -6.6. Recall that 0.6 is the same as 2, so that -6.6 = -6 2 = -20 which is sometimes a more

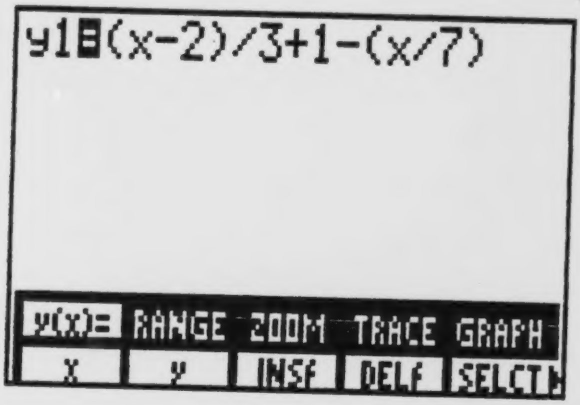
3 3 3

useful form of the solution, depending on the circumstances of the problem.

Example 5: Solve $\frac{x-2}{3} + 1 = \frac{x}{7}$

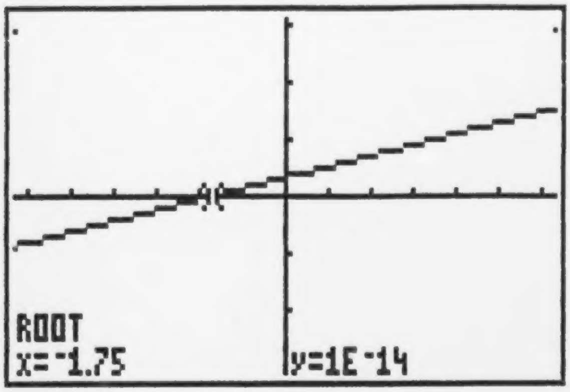
Solution: Graph $y = \frac{x-2}{3} + 1 - \frac{x}{7}$ using ZDECM.

When you type this equation in, be very careful to use parentheses to preserve the order of operations. The calculator follows the same order of operations as we do most of the time. (An exception will be shown in the next example.)



```
y1=(x-2)/3+1-(x/7)
```

Then press TRACE. Notice that the y-values are not "friendly". This does not mean that something is wrong. This happens sometimes when the fractions obtained algebraically are converted to decimals. Press EXIT and MORE. Then press F1 (math) and then F3 (root). Move the cursor near the place on the graph that y = 0. The solution to the equation is x = -1.75 = -7/4.



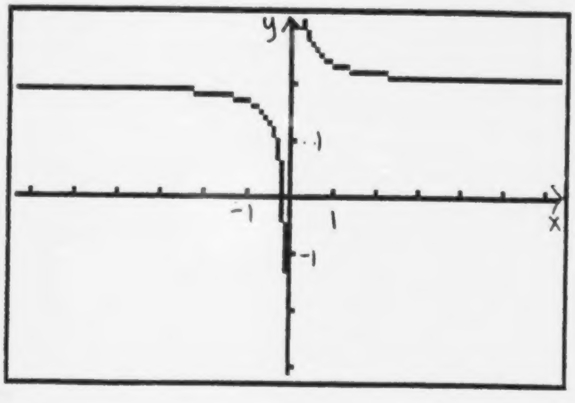
```
ROOT
x=⁻1.75          y=1E⁻14
```

Example 6: Solve $\frac{1}{3}x + 2 = 0$.

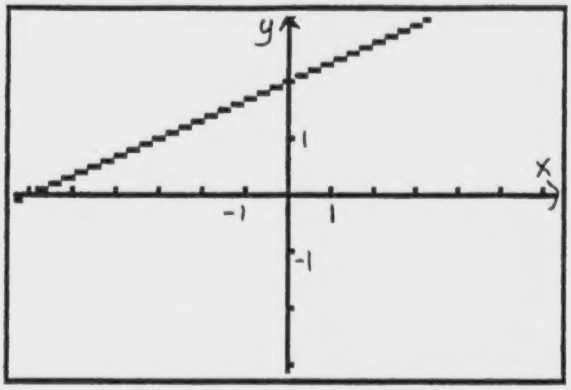
Solution: Now if we were to solve this algebraically instead of on the calculator, we know that the solution should be x = -6. Let us see what happens when we solve this by graphing.

Type in the equation, $y = \frac{1}{3}x + 2$ and graph it using ZDECM.

If you typed in: 1/3x + 2, your graph turned out like this:



If you typed (1/3)x + 2, then your graph turned out like this:



Which one is the correct graph? Check to see which one has the point x = -6, y = 0 on it. It is the second one, isn't it? This is where the calculator does not do the order of operations in quite the same fashion as us. Multiplication and division both have equal "rank" usually and are done in the order that they appear in an expression from left to right. On the calculator though, multiplication happens first no matter what. Most of the time it doesn't matter, but here, where the division must occur first, we must use parentheses in order to guarantee that happens. (Notice that the potential of an error in Example 3 was there if you did not type it in the fashion stated.)

Now we can solve the pharmacy problem that we had at the beginning of this section:

Example 7: Mark has been asked to prepare 500 ml of a 15% dextrose intravenous solution for a premature baby. He has a 70% dextrose solution on hand that he can mix with pure water. How much of each solution does he need to use in order to make the required solution?

Solution: Step 1: What are the unknown quantities?

How much water he needs and how much 70% solution he needs are the unknowns.

Step 2: Relate the unknown quantities with known quantities

some 70% solution + some water = 500 ml of 15% solution

Let x represent the amount of 70% solution. He needs a total of 500 ml of solution. Since x of that is 70% solution, that leaves 500 - x that needs to be water.

Now we have:

x ml of 70% solution + (500 - x) ml of water = 15% solution

The one quantity we know at this point is how much dextrose there is in each of these solutions. 70% of x is dextrose, 0% of 500 - x is dextrose (water has no dextrose in it), and 15% of 500 ml is dextrose. Using this we can now write the sentence (or equation) as:

70% of x + 0% of (500 - x) = 15% of 500

3. Write the equation:

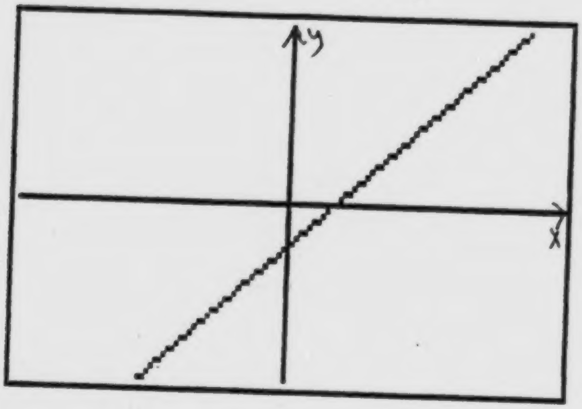
$$.70x + 0(500 - x) = .15(500)$$

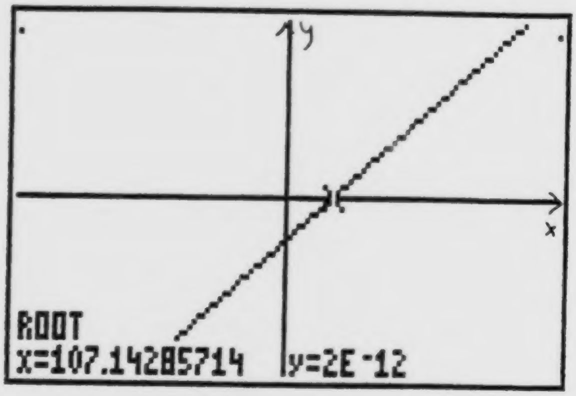
4. Solve the equation:

Graph it. Get everything on one side so that zero is on the other, replace the 0 with y and graph.

$$.70x - .15(500) = y$$

Using ZDECM will not be big enough, so let us ZOOM OUT to make the scale bigger. Remember that our zoom factors are 10 so if we do this twice, then that will increase everything by 100:





```
ROOT
x=107.14285714    y=2E-12
```

Now use ROOT to determine where y = 0. What did you get? What does this mean? You should have gotten x = 107, so that Mark will need to use 107 ml of the 70% solution and 500 - 107 or 393 ml of water in order to make up the 500 ml 15% dextrose solution.

Now you try some problems on your own. If you need to, go back to the example that is listed in parentheses for the problem to get help. When you finish those go on to the review problems. These will test whether you understand what you have learned in this section. You should be able to answer all of the review problems completely before continuing on to the next section.

Exercises for Section 1

Graph the following equations. (See Examples 1,2,3)

1. $y = 4x - 7$
2. $y = 0.81x + 0.2$
3. $y = 0.2x + 18$
4. $2x + 4y = 7$
5. $17x - 2y = 4$
6. $0.3(x - 1) + 2(y + 4) = 15$

Solve by graphing. (See Examples 4,5,6)

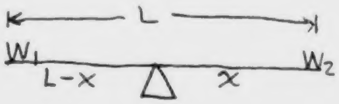
7. $2(x-1) + 4x = 10 - 3x$
8. $13x + 42 = 3$
9. $\frac{1}{4}x - 2 = \frac{17}{4}$
10. $\frac{2x - 1}{3} + 2 = \frac{4x}{5}$

Solve the following problems. (See Example 7)

11. John invested a sum of money for 1 year and earned $920 interest. If 8000 more was invested at 7% interest that was invested at 5% interest, how much did he invest at each rate. (Hint: if x is the amount at 5%, then x + 8000 is the amount invested at 7%.)

12. A farmer mixed gasoline and oil to have two gallons of mixture for his two-cycle chain saw engine. This mixture was 32 parts gasoline and 1 part two-cycle oil. How much gasoline must be added to bring the mixture to 40 parts gasoline and 1 part oil?

13. Suppose you have a uniform beam of length L with a fulcrum x feet away from one end. (See the figure.) If there are objects with weights W_1 and W_2 placed at opposite ends of the beam, then the beam will be balanced if $W_1 x = W_2(L - x)$.



A person weighing 200 pounds is attempting to move a 550-pound rock with a bar that is 5-feet long. Find x.

14. Because air is not as dense at high altitudes, planes require higher ground speeds to become airborne. A rule of thumb is 3% more ground speed per 1000 feet of elevation, assuming no wind and no change in air temperature. (Compute numerical answers to 3 significant digits.)

A) Let V_s = Takeoff ground speed at sea level for a particular plane (in mph)

A = Altitude above sea level (in thousands of feet)

V = Takeoff speed at altitude A for the same plane. (in mph)

Write a formula relating these three quantities.

B) What takeoff ground speed would be required at Lake Tahoe airport (6400 feet) if takeoff ground speed at San Francisco airport (sea level) is 120 mph?

C) If a landing strip at a Colorado Rockies hunting lodge (8500 ft) requires a takeoff ground speed of 125 mph, what would be the takeoff ground speed in Los Angeles (sea level)?

D) If the takeoff ground speed at sea level is 135 mph and the takeoff groundspeed at a mountain resort is 155 mph, what is the altitude of the mountain resort in thousands of feet?

Review Problems for Section 1

1. What is the window needed to show the x- and y-intercepts of the graph of $y = 4x + 18$?
2. Graph the equation $10x + 15y = 38$ in a window that gives a complete graph.
3. State the x- and y-intercepts for problems 1 and 2. Use ZOOM or ROOT when necessary.
4. A fuel oil distributor has 120,000 gallons of fuel with 0.9% sulfur content, which exceeds pollution control standards of 0.8% sulfur content. How many gallons of fuel oil with a 0.3% sulfur content must be added to the 120,000 gallons to obtain fuel oil that will comply with the pollution control standards?

Section 2

Graph the following equations in a window that gives a complete graph. As you do, make a quick sketch next to each one that shows the basic shape of the graph. In some of the equations below, you will need to solve for y first.

A. $2x + 3y = 5$

B. $\frac{2}{x} + 3y = 5$

C. $5x^2 + 2x + 1 = y$

D. $-8x + 9y = -17$

E. $6(x + 1) - 2 = y$

F. $2x^2 + 3y = 5$

G. $2x^3 + 3y = 5$

H. $3x - 4y = 7$

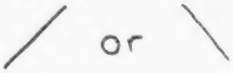
I. $\frac{6}{x + 1} - 2 = y$

J. $6(x + 1)^2 - 2 = y$

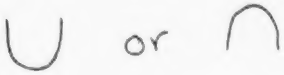
What do you notice about the shapes of the graphs? Do you notice any patterns? In particular, do you notice any relationship between the type of equation and its graph? If so state your speculation.

Hopefully, your graphs had the following shapes:

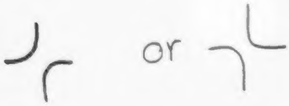
A, H, D, E -



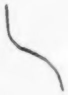
F, C, J -



B, I -



G -



Notice that the equations whose graphs were lines (A, H, D, E)

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were all first degree equations. In other words, the variables were all only raised to the first power. (Recall that if a variable is in the denominator then it would be written with a negative exponent in the numerator.) These first degree equations are special since their graphs are lines:

Definition: A linear equation in 2 variables has the form $ax + by + c = 0$, where a and b are not both zero at the same time.

Theorem: The graph of a linear equation in two variables is a line.

By inspection, not by graphing, which of the following equations will have graphs that are lines?

a. $4(x - 1) + 2y = 7$ b. $y = 2x^2 + 4x + 1$

c. $0.37x + 0.73(20 - x) = y$ d. $2x^2 + 8 - 3y = 2$

e. $0.37x^2 + 0.73(20 - x)^2 = y$

Now check yourself by graphing them on the calculator. Be sure to find a complete graph of each one before you draw a conclusion.

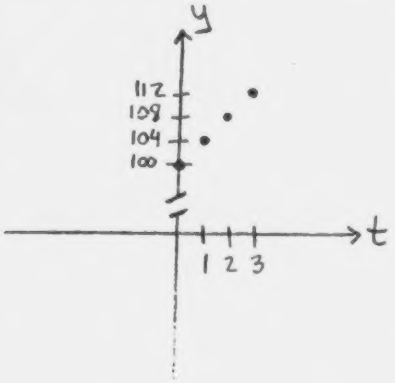
Lines have some special features which are important in application problems. In Section 1, we saw that finding the x-intercepts of a graph helped us solve the pharmacy problem.

Consider the following situation:

On January 1, Alice deposits $100 in a savings account at the bank. From then on, her deposits and the interest that she is earning increase the amount in her account by $4 every month. How much is she going to have in her account on December 31?

The amount in Alice's account depends on the time that has elapsed since she first deposited her $100. Plot a few points to show how much she has in the account after 1 month, 2 months, 3 months.

At the beginning, t = 0, she only has $100. This gives the point (0,100). After 1 month she has 4 more dollars, so now the point is (1, 104). After two months, the point is (2, 108). After three, it is (3,112).



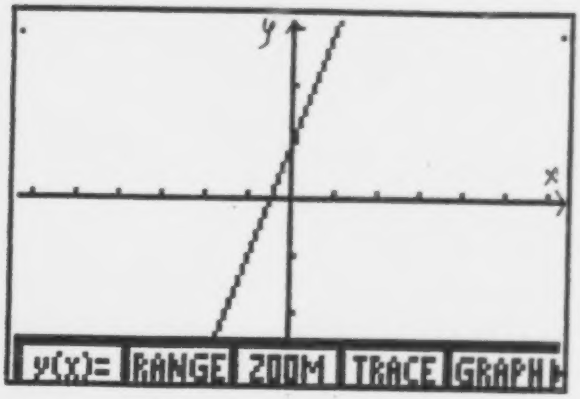
What do you notice about the points? If you were to connect them, they would form a straight line. If we move horizontally from one point to the next point, we notice that the change in t is 1. If we move vertically from one point to the next the vertical change is 4. The ratio of these two numbers, vertical change to horizontal change, is 4 to 1, or 4. This ratio represents a rate of change, and is called the slope of the line.

Definition: $$\text{slope} = \frac{\text{vertical change in graph}}{\text{horizontal change}}$$

We use the letter m to represent the quantity of slope. Since the y-values are vertical and the x-values are horizontal, we may write:

$$m = \frac{\text{change in } y}{\text{change in } x}$$

Consider the equation $y = 2x + 1$. Graph it on your calculator.



Now pick two points off the line and compute the difference in the x-values and in the y-values: For example:

first point (0, 1), second point (-1, -1)

change in x = second x - first x = -1 - 0 = -1

change in y = second y - first y = -1 - 1 = -2

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{-2}{-1} = 2$$

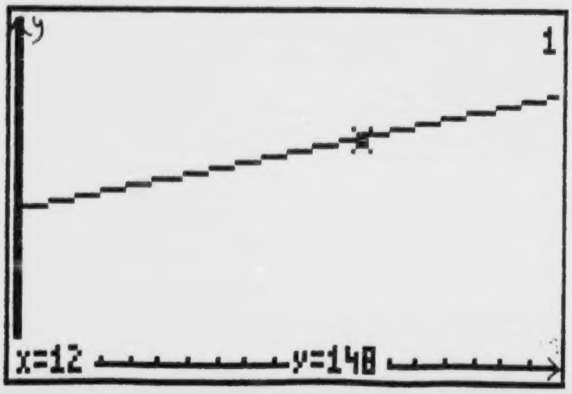
Pick any other two points off the graph and compute the slope again. Notice that you always get the same slope value. This is because a line has the same slope all along it.

Notice also that the y-intercept of the graph is (0, 1). Now look at the equation: $y = \mathbf{2}x + \mathbf{1}$. The coefficient of the x in the equation, 2, is the slope and the constant, 1, is the y-value of the y-intercept. This form of the equation is called the **slope-intercept form** because we can read the slope and the y-intercept straight from the equation.

Theorem: The slope-intercept form of the equation of a line is $y = mx + b$, where m is the slope and b is the y-value of the y-intercept.

Now that we have a general equation of a line, we may find equations of lines provided we have two points or one point and the slope.

Now go back to the problem with Alice. We still need to know how much money will be in her account on December 31. In order to do this we need to find a formula or equation of the line that represents the amount of her money. The slope is 4 from above, and we know the y-intercept, the amount of money she has at $t = 0$. (What is it?) Placing those values into the above equation, we have $y = 4t + 100$. Now we may either graph the equation on the calculator and TRACE to where $t = 12$ (after 12 months is when we are asking for) or we may substitute directly into the equation and determine that she will have $148 in the account.



$y = 4(12) + 100$
$y = 48 + 100$
$y = \$148$

Based on the slope formula and knowing one point on a line, there is another useful formula for finding the equation of a line:

Theorem: Given the slope, m, of a line and a point (x_1, y_1) on the line, the **point-slope form** of the equation of the line is $y - y_1 = m(x - x_1)$.

Example 3: For the Big Company, the relationship between the number of units sold and the profit is linear. If the profit is \$500 when 300 units are sold and \$3500 when 900 units are sold, write the equation relating profit y to units sold x and find a sensible graph for the problem.

Solution: The information given says the relationship is linear. That means that if we plot the points, they lie on a line.

The points are (300, 500) and (900, 3500). From these we may compute the slope:

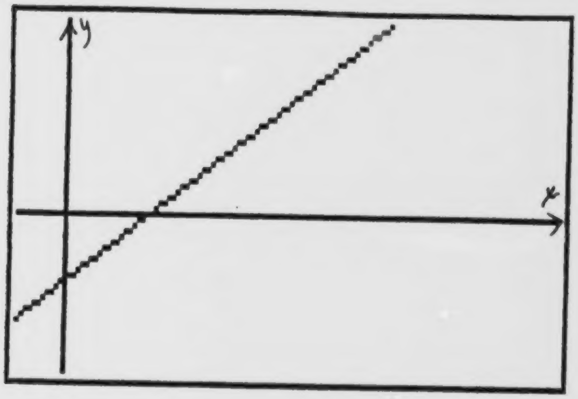
$$m = \frac{3500 - 500}{900 - 300} = \frac{3000}{600} = 5$$

Using that and one of the points above, say (300,500), in the point-slope form of the equation of a line:

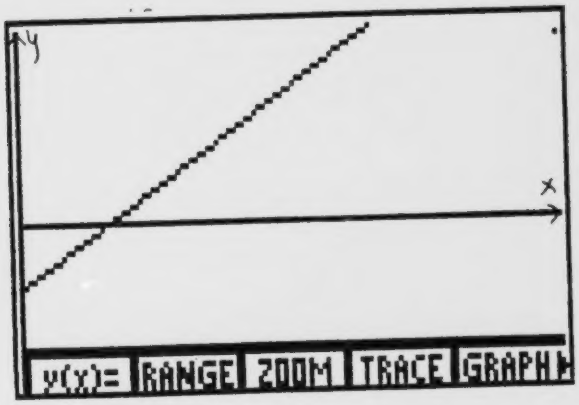
$$y - 500 = 5(x - 300)$$

$$y = 5x - 1000.$$

When we graph this equation we get



The slope of this graph is $5 per unit sold. That is the rate of increase of the profit for each unit that is sold. Does it seem reasonable that there should be negative numbers for x? No. Thus, when we draw a graph for this problem we will not include where x is negative.



Example 4: Suppose an airplane is coming in for landing at DFW

Airport and it is at an altitude of 35000 feet at 12:00. At 12:02 it is at 30000 feet and at 12:05 it is at 22500 feet. What is its rate of descent?

Solution: Let 12:00 be considered time $t = 0$. Then we have the point $(0, 35000)$. At time $t = 2$, we have $(2,30000)$. Thus the slope or rate of descent is $m = \frac{35000 - 30000}{0 - 2} = \frac{5000}{-2} = -2500$ feet per minute.

Now does the plane's rate of descent remain the same? To determine this we will find the equation of the line containing the first two points and then determine if the third point lies on that line. Note that $(0,35000)$ is the y-intercept, so using the slope-intercept form of the equation of the line, we have:

$$y = -2500x + 35000$$

Using the point $y = 22500$ when $x = 5$:

$$22500 = -2500(5) + 35000$$

$$22500 = -12500 + 35000$$

$$22500 = 22500.$$

Thus the plane continues to descend at the same rate for awhile.

Now you try some problems on your own. If you need to, go back to the example that is listed in parentheses for the problem to get help. When you finish those go on to the review problems. These will test whether you understand what you have learned in this section. You should be able to answer all of the review problems completely before continuing on to the next section.

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Exercises for Section 2

1. Robert wants to build a rabbit hutch. If he uses 68 square feet of plywood, the cost is $26. If he uses 88 square feet of plywood, the cost is $31.
   a) What is the cost per square foot to build the hutch? (Example 2)
   b) Find the equation of the line that expresses the total cost y in terms of the number of square feet of plywood x used.(Example 1)
   c) How much will it cost Robert if he uses 125 square feet? (Round to the next higher dollar, if necessary.)
2. It is Christmas Eve at Macy's. At the beginning of the day, there were 100 teddy bears in stock. From the moment the store opened, the teddy bears were selling at a rate of 15 every 2 hours. Write an linear equation relating the number of teddy bears left in the store, y, and the amount of time x that has elapsed since the store opened. When will they run out of teddy bears? (Example 1)

Review Problems for Section 2

1. Which of the following are lines? (Preliminary examples)

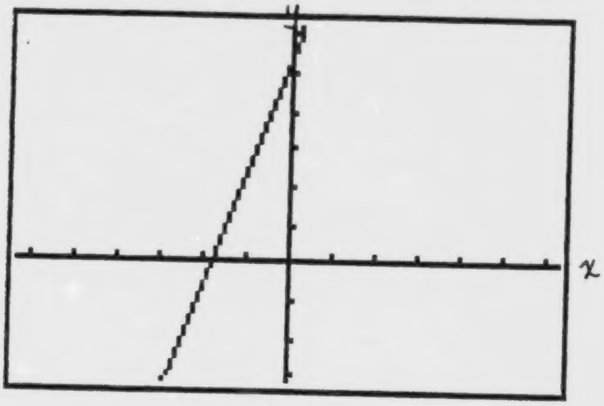
   a) $0.2x + 3y = 17$ b) $4x^2 - 3x = 2y$

   c) $x - 0.1x^2 = x^3 + y$

2. Find the slope of the line that passes through the points (9,13), and (21, -24).

3. Find the equation of the line that passes through the points (9, 13) and (21, -24).

4. Find the equation of the following line:



5. Suppose that the relationship between the public demand for a particular brand of toaster oven and its unit price is linear. If the price of the oven is set at $50, the demand is 100 toaster ovens. If the price of the oven is set at $60, the demand is 70 toaster ovens. Use the slope to determine the rate at which the demand for toaster ovens is decreasing with respect to price. Write a sentence explaining the real-world significance of this number.

Section 3

Sometimes in application problems it is easier to set up two relationships or equations to represent a situation than just to use one. In this case, we would like to be able to solve a system of equations on the calculator.

Example 1: John goes to the post office to buy stamps for some postcards and some letters that he wants to mail. The stamps for postcards are 20 cents and the stamps for letters are 32 cents. John has six more letters than postcards. The total bill for his stamps is $9.20. How many postcards and letters did he send?

Solution: Recall the problem solving steps from section 1. The first step is to identify the unknown quantities:

We do not know: 1. how many stamps he bought,
2. how many letters he mailed,
3. how many postcards he mailed.

Step 2: Find a relationship between known quantities and unknown quantities:

a) number of letters is 6 more than number of postcards

b) cost to mail letters + cost to mail postcards is $9.20.

If we let x be the number of letters, and y be the number of postcards, then we have:

x is 6 more than y for a) above.

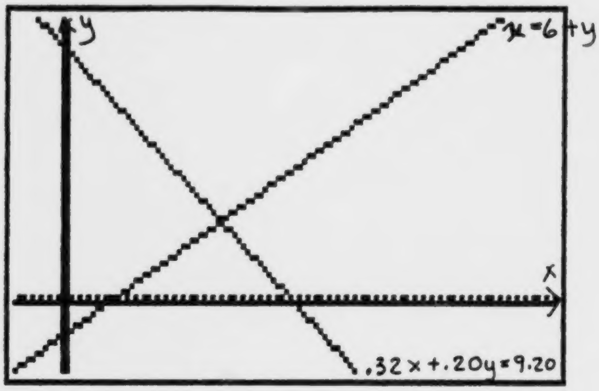
The cost to mail letters is 32 cents per letter, so total cost for letters is $.32x$. Likewise the cost for a postcard is 20 cents, so the total cost for the postcards is $.20y$.

So for b) we have: $.32x + .20y$ is 9.20

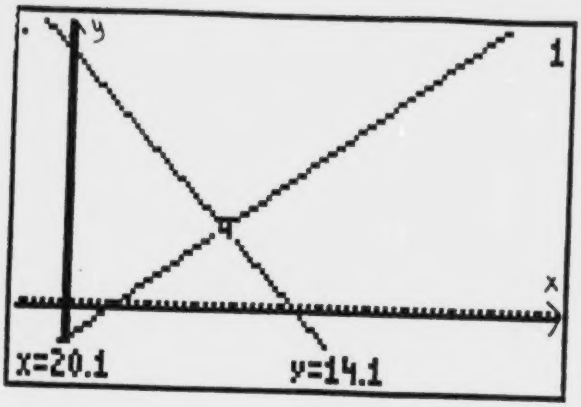
Step 3: Set up equations. We almost have these. From step 2, we now have $x = 6 + y$

and $.32x + .20y = 9.20$

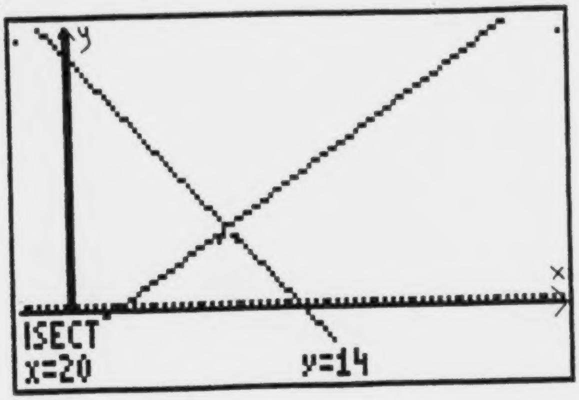
Step 4: Solve the system. Graph these equations on the grapher. Be sure to solve for y in each case.



The solution will be the values of the variables at the point where the two lines intersect. This is because both equations will be satisfied by that point. We may either find that point by tracing (in the same fashion as in section 1 and 2 to determine the values) or by using the ISECT function on the calculator. We will use ISECT in this example.



To get to ISECT on the calculator, from the basic graph menu, press MORE. At F1 is Math. Press it. Then press MORE again. At F4 is ISECT. Press this. The cursor will be on the first curve. Press enter. Then the cursor is flashing on the second curve. The machine is essentially asking you if this is the curve you want to find the intersection with. Press Enter. Now it will compute the intersection point.



Hence, he mailed 14 postcards and 20 letters.

Notice that had we substituted the expression $x - 6$ into the second equation for y and we solved that equation on the calculator, then we would have found the x - intercept which is the number of letters. Then we would have gone back and solved for the number of postcards. Either way is satisfactory, however, if we already have two equations set up, it is easier to just find their point of intersection.

Example 2: How many grams of an alloy containing 15% silver must be melted with 40 grams of an alloy containing 6% silver to obtain an alloy containing 10% silver ?

Solution: Let x = number of grams of 15% silver and let y = total amount of 10% silver.

Then amount of silver in 15% silver + amount of silver 6% silver = amount of silver 10% silver

$$0.15x + 0.06(40) = 0.10y$$

Also, amount of 15% silver + amount of 6% silver = amount of 10%.

$$x + 40 = y$$

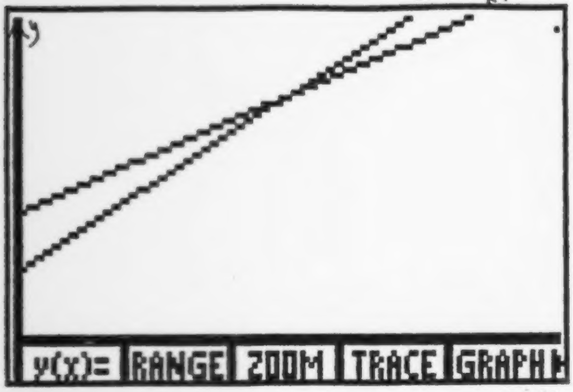
This gives us two equations: $.15x + .06(40) = .10y$

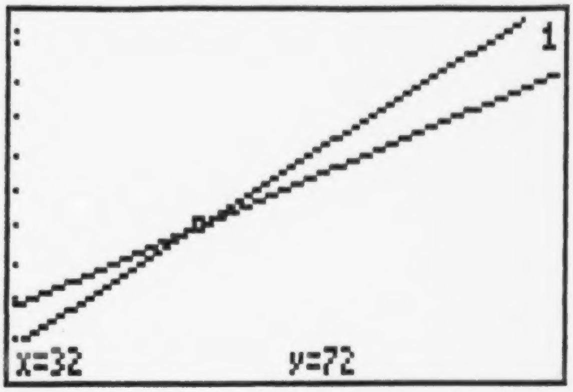
$$x + 40 = y$$

Graph both of these and use Trace and Zoom to find the intersection point.

$.15x + .06(40) = .10y$

$x + 40 = y$





x=32 y=72

Thus, 32 grams of the 15% silver are needed.

Sometimes when we graph two equations, they look like they are parallel. In order to determine if they are, it is helpful to know that two lines are parallel if their slopes are the same. Thus by comparing the slopes of two lines, we will know whether our system has a solution.

Example 3: Determine if the lines containing the following points are parallel.

a) line 1: (9,8), (5,-3); line 2: (4,3), (6,5)

b) line 1: (7.5, 3.6), (1.23, 4.35); line 2: (6.27,1), (0,1.75)

Solution: To determine if the two lines are parallel, compute their slopes and compare.

a) line 1: $m = \frac{-3-8}{5-9} = \frac{-11}{-4} = \frac{11}{4}$; line 2: $m = \frac{5-3}{6-4} = \frac{2}{2} = 1$

$$\frac{11}{4} \neq 1$$ therefore line 1 is not parallel to line 2.

b) line 1: $m = \frac{4.35-3.6}{1.23-7.5} = \frac{.75}{-6.27} = \frac{-25}{209}$;

line 2: $m = \frac{1.75-1}{0-6.27} = \frac{.75}{-6.27} = \frac{-25}{209}$

The slopes are the same thus the lines are parallel.

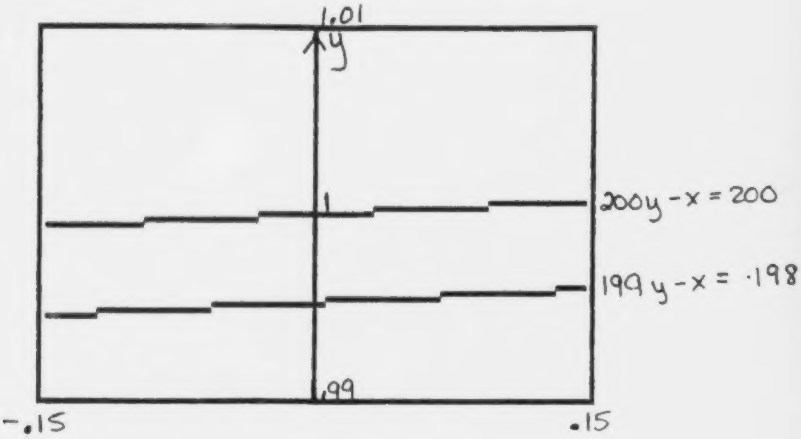
Now you try some problems on your own. If you need to, go back to the example that is listed in parentheses for the problem to get help. When you finish those go on to the review problems. These will test whether you understand what you have learned in this section. You should be able to answer all of the review problems completely before continuing on to the next section.

Exercises for Section 3

1. A chemical technician combines a 20% acid solution with a 40% acid solution to obtain 12 liters of a 25% solution. How many liters of the 20% solution and how many liters of the 40% solution should he use? (Example 2)
2. A total of $12000 is invested in two corporate bonds that pay 10.5% and 12% interest. The annual interest is $1380. How much is invested in each bond? (Example 1)
3. The graphs of the two equations appear to be parallel. Yet, when the system is solved algebraically, it has a solution. Find the solution on the calculator and explain why it does not appear on the portion of the graph below: (Example 3, Example 1)

$200y - x = 200$

$199y - x = 198$



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Review Problems for Section 3

1. Solve the system using Trace and Zoom: $2x + 3y = 7$
   $5x - 2y = 18$

2. Solve using ISECT: $3x - 4y = 8$
   $0.1x + 2y = 1.47$

3. Which of the following lines are parallel?

   a) $3x + 6y = 7$ b) $4y - 8x = 5$

   c) $y - 3 = \frac{2}{7}(x - 4)$ d) $y - 5 = -\frac{1}{2}(x + 8)$

4. Suppose you are the night manager of a shoe store. On Saturday night you are going over the receipts of the previous week's sales. Two hundred forty pairs of tennis shoes were sold. One style sold for \$66.95 and the other sold for \$84.95. The total receipts were \$17,652. The cash register that was supposed to record the number of each type malfunctioned. Can you recover the information? If so, how many shoes of each type were sold?

Section 4

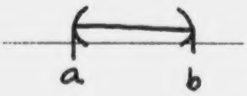
Recall that when we want to graph on the calculator, we must set the range that will be shown on the graphing screen. These limits that we set for x and y may be written using inequality notation:

$\text{xmin} \leq x \leq \text{xmax}$ and $\text{ymin} \leq y \leq \text{ymax}$.

There is a special notation that can be used to describe this called interval notation. The descriptions of different intervals are described in the below:

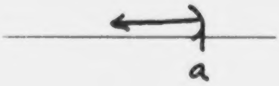
$a < x < b$

(a, b)



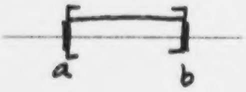
$x < a$

$(-\infty, a)$



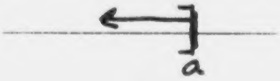
$a \leq x \leq b$

$[a, b]$



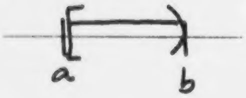
$x \leq a$

$(-\infty, a]$



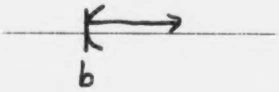
$a \leq x < b$

$[a, b)$



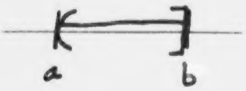
$b < x$

(b, ∞)



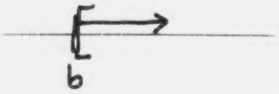
$a < x \leq b$

$(a, b]$



$b \leq x$

$[b, \infty)$



Example 1: Write the inequality in interval notation: $-3 \leq x < 7$

Solution: $[-3, 7)$

We have seen that we can solve equations on the calculator. It is possible to solve inequalities on the calculator as well. Recall that if a number is bigger than zero then it is positive, and if a number is smaller than zero then it is negative. These facts will be useful.

Example 2: Suppose Sammie the ice cream lady would like to make a profit of at least $125 each afternoon that she drives her truck. It costs her $45 per outing to fill up her freezer and buy gas. She can sell her ice cream at an average of 62 cents per ice cream treat. How many ice cream treats much she sell to make her minimum profit?

Solution: Unknowns: how many ice creams she needs to sell

Find relationships and set up equations/inequalities:

receipts from ice cream sold: .62 per ice cream

total cost per day: $45

profit = total receipts - cost

Total receipts from ice cream are .62 times the number of ice creams. Let the number of ice creams be x, then we have $.62x$. Profit must be at least $125. So this gives the inequality:

$$125 \leq .62x - 45$$

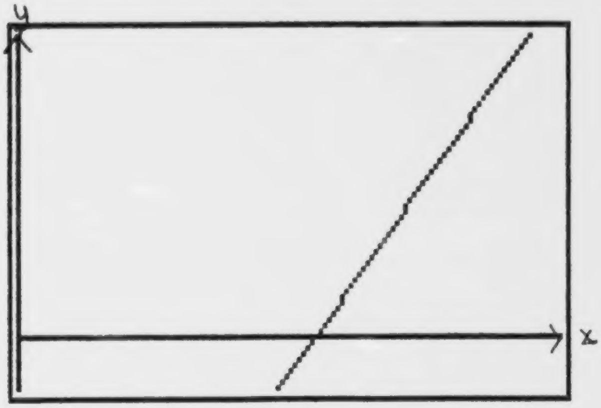
Solve: To solve this on the calculator, we need to have 0 on one side and then we will set up an equation with y to graph. The variable y will represent the expression with x in it and we will be looking for where that is bigger than 0 on the graph.

$0 \leq .62x - 45 - 125$

Graph $y = .62x - 45 - 125$. The y-value needs to be at least 170 (45 + 125) and the x value will need to be large. Use

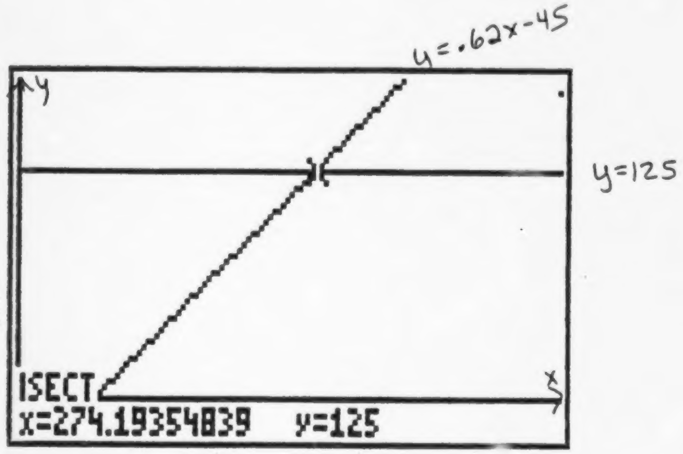
ymax = 62 x 2
xmax = 126 x 4
ymin = -20
xmin = 0

The ymin at -20 will allow room for the menu at the bottom of the screen without covering up part of the graph. Remember both variables need to be positive for the problem to make since. Also, set the xscl and yscl both to 0. This eliminates the "graph paper"-like hash marks on the axes.



Where is this equal to zero? Use root to find that point. This means that as long as she sells at least 274.194 ice creams, she will make at least $125. Since it is not reasonable to expect her to sell a piece of an ice cream treat, then she must sell at least 275 ice creams. Notice that when x is 275, the y value is greater than zero (in fact, it is 0.5. Get this by zooming in twice around x = 275 until the x-value in TRACE is exactly 275). Thus her profit will be slightly greater than $125.

Another way to solve this problem is by graphing $y = .62x - 45$ and $y = 125$ and looking for where the first graph is above the second graph.



```
ISECT
x=274.19354839    y=125
```

Notice that we still have the same solution as before by using ISECT with two graphs.

Example 3: Tim needs at least an average of 90 to get an A in English. There are four tests and his first three test scores were 92, 78, and 94. What is the minimum score that he can make on the fourth test and still have an A?

Solution: Let x be the score on the fourth test.

To find an average, add the scores together and divide by the number of scores. This average must be at least 90.

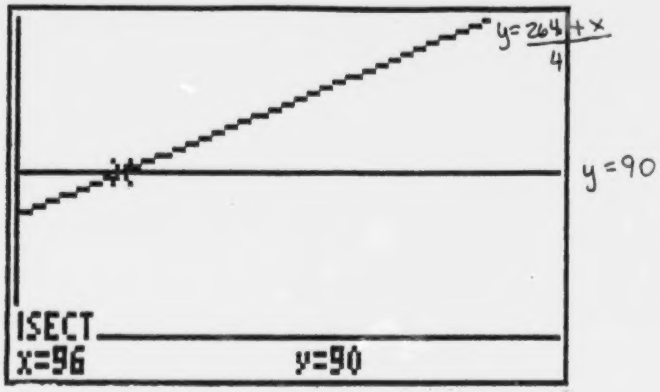
$$\text{average} \geq 90$$

$$\frac{92 + 78 + 94 + x}{4} \geq 90$$

Thus we need to solve $\frac{264 + x}{4} \geq 90$

Graph $y = \underline{264 + x}$ and $y = 90$. Don't forget parentheses.

Where is the graph of the first equation above that of the second?



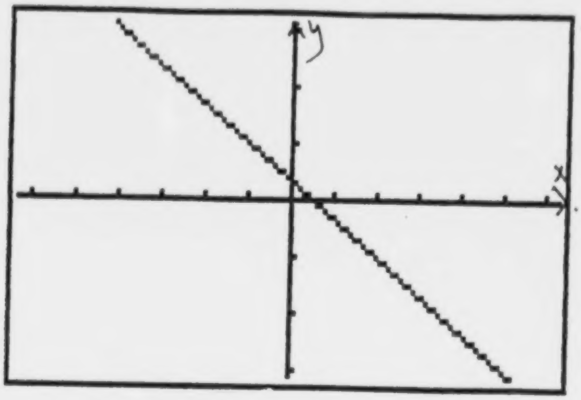
It is above the line y = 90 on the interval [96, ∞), in other words when $x \geq 96$. So, Tim must make at least a 96 in order to make an A in English.

Sometimes it is necessary to consider inequalities with two variables. For instance, how many screwdrivers and hammers can be made if it takes a screwdrivermaker 2 hours to make a screwdriver and a hammermaker 1.5 hours to make a hammer, and no more than a total of 74 hours per week can be worked?

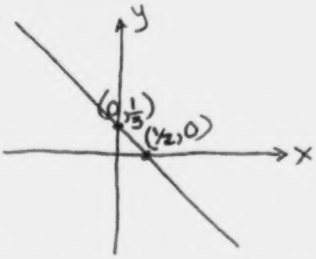
Suppose x is the number of screwdrivers and y is the number of hammers. Then it takes a total of 2x hours to make x screwdrivers, and a total of 1.5y hours to make y hammers. Together these times must be less than 74. Thus, $2x + 1.5y \leq 74$. In order to solve this, we will need to examine the method we will use with the calculator.

Example 4: Solve $2x + 3y \geq 1$

Solution: First we will graph the equation $2x + 3y = 1$. Recall that you must solve for y first.



Sketch the graph on your paper, showing the x and y intercepts.



We want the set of points that satisfy the inequality. These will lie on either one side of the line or the other. To see how this is true, let us examine a few points. Use your cursor (do not use trace) to pick out some points above the line and then pick out some points below the line. Test these points in the original inequality. Which ones created true statements? Which ones didn't? Can you draw a conclusion about where the points came from that worked? They all came from the same side, the side above the line. Thus we shade that side of the line. All the points on that side, when placed into the inequality, will yield a true statement.

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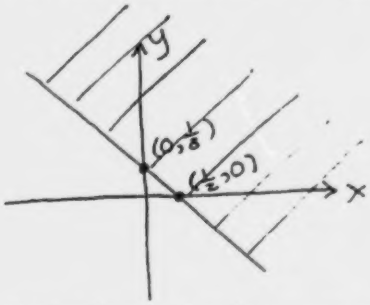
That is the solution of that inequality.



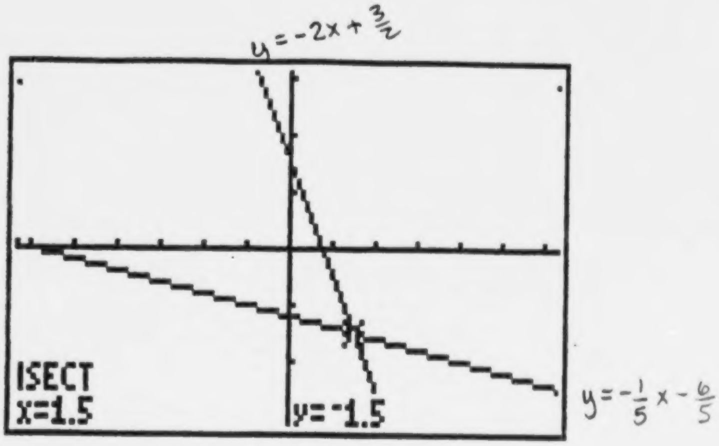
Sometimes it is necessary to solve a system of inequalities. The same principle applies to doing this as was used to solve systems of equations: we need to find the points which satisfy both equations or inequalities. Systems of inequalities arise in solving constraint problems in business as we will see later.

Example 5: Solve the system $4x + 2y \geq 3$

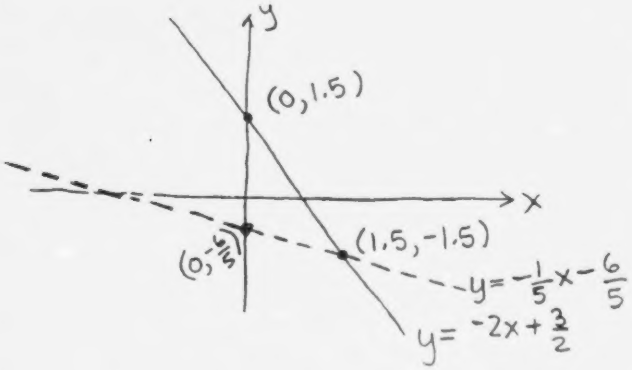
$$x - 5y < 6$$

Solution: In order to solve this system, we need to find the points that satisfy both inequalities. This will occur where the shaded areas for each inequality overlap.

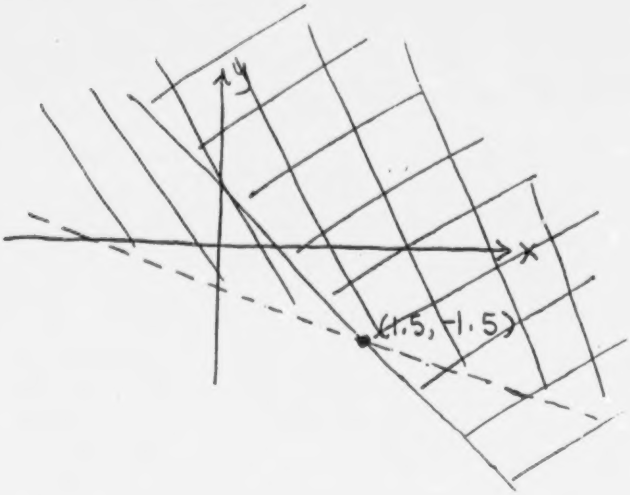
Graph each of the inequalities as equations on the calculator and determine where they intersect. This point is called a corner point.


y = -2x + 3/2
ISECT
x=1.5
y=-1.5
y = -1/5 x - 6/5

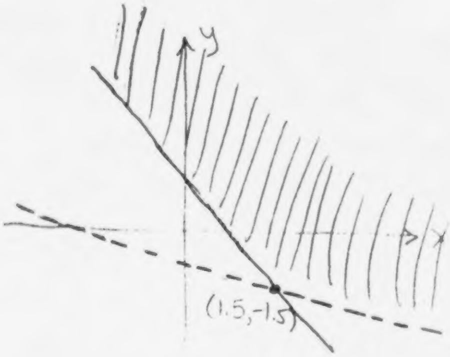
Make a hand-drawn sketch of this then we will test a point on one side of each line to determine the side to shade.


y
(0,1.5)
x
(1.5,-1.5)
(0,-6/5)
y = -1/5 x - 6/5
y = -2x + 3/2

Notice that the first line is drawn with a solid line. This is because the boundary of the region to be shaded (the line itself) is included in the set of points that satisfy the inequality. The second line is broken because it is the boundary, however its points do not get included. (Try some of the points on the lines in each of the inequalities. Do they satisfy the inequality? If so, draw it with a solid line. If not, use a broken one.)



The portion of the shaded region where they overlap represents the points that satisfy both equations. So the final solution of the system is:



Now you try some problems on your own. If you need to, go back to the example that is listed in parentheses for the problem to get help. When you finish those go on to the review problems. These will test whether you understand what you have learned in this section. You should be able to answer all of the review problems completely before continuing on to the next section.

Exercises for Section 4

1. Find the solution graph of the screwdriver and hammer problem on page 43. (Example 4)
2. Graph the solutions of the following inequalities. (Example 4)
   a) $2x - 2y \geq 5$
   b) $y \leq 0.31x - 4$
   c) $2y - 3 > x$
3. Solve the systems. State and label the corner points. (Example 5)
   a) $2x + y > 5$
      $x + 2y > 4$
   b) $3x + 3y \leq 6$
      $x - 2y > 2$
4. A manufacturing company makes two types of water skis: a trick ski and a slalom ski. The trick ski requires 6 labor-hours for fabricating and 1 labor-hour for finishing. The slalom ski requires 4 labor-hours for fabricating and 1 labor-hour for finishing. The maximum labor-hours available per day for fabricating and finishing are 108 and 24, respectively. If x is the number of trick skis and y is the number of slalom skis produced per day, write a system of inequalities that indicates appropriate restrictions on x and y. Find the solution of the system graphically. (Example 5)

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Review Problems for Section 4

1. Solve $2x - 3y \geq 5$
2. Solve $4(x-2) + 3 \leq -2x + 7$
3. Solve the system $2x - 3 < 2y$ and state the corner points.

   $3y - 4 < 2x$
4. For a business to make a profit it is clear that revenue R must be greater than cost C; in short, a profit will result only if $R > C$. If a company manufactures records and its cost equation for a week is $C = 300 + 1.5x$ and its revenue equation is $R = 2x$, where x is the number of records sold in a week, how many records must be sold for the company to realize a profit?

Section 5

In the previous section we saw how to set up problems that described business situations. For example in exercise 4 in section 4 these inequalities are called constraints. Constraints that are always satisfied are called natural constraints. What would those be for that exercise? Since a negative number of skis cannot be produced, the natural constraints would be $x \geq 0$ and $y \geq 0$.

Example 1: Bruce builds portable buildings. He uses 10 sheets of plywood and 15 studs in a small building, and he uses 15 sheets of plywood and 45 studs in a large building. Bruce has available only 60 sheets of plywood and 135 studs. If Bruce makes a profit of $400 on a small building and $500 on a large building, how many of each type of building can Bruce make to maximize his profit?

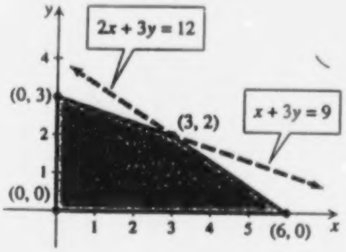
Solution: First we will set up the inequalities that describe Bruce's constraints.

The natural constraints are $x \geq 0$ and $y \geq 0$ where x is the number of small buildings he can make and y is the number of large buildings he can make. Since he only has 60 sheets of plywood, we have $10x + 15y \leq 60$. Since he only has 135 studs, we have $15x + 45y \leq 135$. Thus the constraints are:

$$x \geq 0, \; y \geq 0$$
$$10x + 15y \leq 60$$
$$15x + 45y \leq 135$$

The equation for his total profit gives us what is called the objective function: $P = 400x + 500y$.

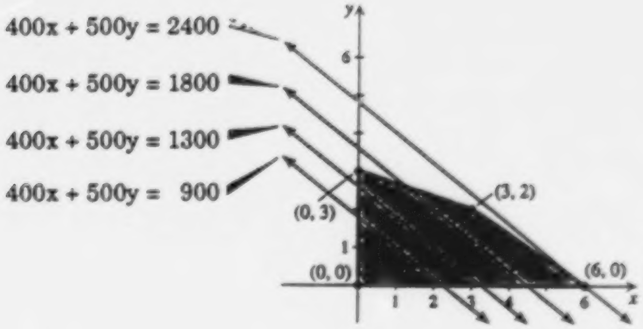
Graph the set of constraints. The solution of this system is called the feasible solution. Only points in this region will satisfy all of the constraints, and hence will be likely to yield a maximum profit.



Bruce is interested in the maximum profit, subject to the constraints on x and y. Suppose x = 1 and y = 1; then the profit is $P = 400(1) + 500(1) = \$900$.

In fact, the profit is $900 at any point on the line $400x + 900y = 900$. (check that- plug in some points on that line).

The profit is $1300 at any point on the line $400x + 500y = 1300$ and it is $1800 at any point on the line $400x + 500y = 1800$. The graphs of these lines are shown below. Notice that the larger profit is found on the higher profit line and all of the profit lines are parallel.(Why?) Bruce wants the highest profit line that intersects the region of feasible solutions. Notice that in the picture below that the highest profit line that intersects the region and is parallel to other profit lines will intersect the region at the corner (or vertex) (6,0).



Thus he should build 6 small buildings and no large buildings in order to maximize his profit. His maximum profit will be P = 400(6) + 500(0) = $2400.

Sometimes we may want to minimize a linear equation, like a cost function. In general though, if a maximum or minimum value exists, then the maximum or minimum value of a linear function subject to linear constraints occurs at a corner point or vertex of the region determined by the constraints.
It is possible for the maximum or minimum to occur at more than one of the corners, and hence at every point on the line segment that joins them.

To use this new procedure on Bruce's buildings, we will try each of the corners in the profit equation. The corners are (0, 0), (6, 0), (0, 3), and (3, 2).
P(0,0) = 400(0) + 500(0) = 0 (minimum profit)

$P(6,0) = 400(6) + 500(0) = 2400$ (maximum profit)

$P(0,3) = 400(0) + 500(3) = 1500$

$P(3,2) = 400(3) + 500(2) = 2200.$

From this list the maximum profit is $2400 when 6 small and 0 large buildings are built. The minimum occurs when he makes no buildings at all.

Now you try some problems on your own. If you need to, go back to the example that is listed in parentheses before the problem to get help. When you finish those go on to the review problems. These will test whether you understand what you have learned in this section. You should be able to answer all of the review problems completely before continuing on to the next section.

Exercises for Section 5

All of these will follow example 1.

1. One serving of Muesli breakfast cereal contains 4 grams of protein and 30 grams of carbohydrates. One serving of Multi Bran Chex contains 2 grams of protein and 25 grams of carbohydrates. A dietitian wants to mix these two cereals to make a batch that contains at least 44 grams of protein and at least 450 grams of carbohydrates. If the cost of Muesli is 21 cents per serving and the cost of Multi Bran Chex is 14 cents per serving, then how many servings of each cereal would minimize the cost and satisfy the constraints?

2. At Taco Town a taco contains 2 oz of ground beef and 1 oz of chopped tomatoes. A burrito contains 1 oz of ground beef and 3 oz of chopped tomatoes. Near closing time the cook discovers that they have only 22 oz of ground beef and 36 oz of tomatoes left. The manager directs the cook to use the available resources to maximize their revenue for the remainder of the shift. If a taco sells for 20 cents and a burrito for 65 cents, then how many of each should they make to maximize their revenue, subject to the constraints?

3. Kimo's Material Company hauls gravel to a construction site, using a small truck and a large truck. The carrying capacity and operating cost per load for the small truck are 20 cubic yards and $70, respectively. The carrying capacity and operating costs for the large truck are 40 cubic yards and $60, respectively. Kimo must deliver a minimum of 120 cubic yards

per day to satisfy his contract with the builder. The union contract with his drivers requires that the total number of loads per day be a minimum of 8. How many loads should be made in each truck per day to minimize the total cost?

4. Tina's telemarketing employs part-time and full-time workers. The number of hours worked per week and the pay per hour for each is given in the table below. Tina needs at least 1200 hours of work done per week. To qualify for certain tax breaks, she must have at least 45 employees. How many part-time and full-time employees should be hired to minimize Tina's weekly labor cost?

	Part-time	**Full-time**
Hr/wk	20	40
Pay/hr	$6	$8

Section 1:

#4Review and #14 exercises come from Barnett and Zeigler

#12 and #13 Exercises come from Larson.

Section 5 problems from Dugopolski's College Algebra

Quadratic Equations and Inequalities

APPLIED ALGEBRA CURRICULUM MODULE

Objectives

Section 1: After completing this section, you will be able to:

- determine if an equation represents a parabola
- determine the equation of a parabola that has been translated
- identify the vertex, maximum or minimum value, and intercepts of a parabola

Section 2: After completing this section, you will be able to:

- solve a quadratic equation using the quadratic formula
- interpret the solution from the quadratic formula as it applies to real situations
- set up quadratic equations in applied problems
- interpret the graph of a quadratic as it applies to real situations

Section 3: After completing this section, you will be able to:

- solve quadratic inequalities using the graphing calculator
- set up quadratic inequalities in applied problems
- interpret the solutions of quadratic inequalities in real situations

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Section 1

Graph the following equations on the graphing calculator. Record their basic shapes on your paper.

A. $y = 4x^2$
B. $y = -x^2 + 6x$
C. $y = -x^2 - 2x - 3$
D. $2y + 4x = -x^2 + 2$
E. $y = (x - 3)^2$
F. $y = x^2$

Make sure that you find a complete graph of each one.

What did you notice about the graphs? Did they all seem to have the same shape?

Now graph the following equations:

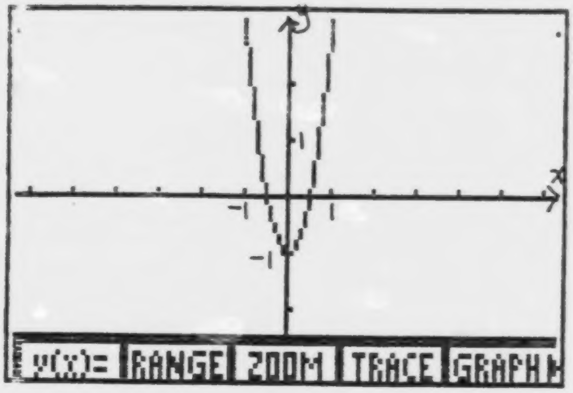
1. $y = 2x + 3$
2. $y = 3x^2 - 4x + 7$
3. $y = 2x^2 + 3$
4. $y = 3x^3 - 4x + 7$
5. $y = 2x^3 + 3$
6. $y = 3x - 4$

Did any of these give the same shapes as the first set? Which ones?

We may make a generalization at this point about the graphs that we have seen.

Definition: An equation of the form $y = ax^2 + bx + c$ is called a quadratic equation and its graph is called a parabola.

Consider the graph of $y = 4x^2 - 1$. Put it on your calculator. You should have the following:



Notice that it crosses the x-axis at (.5,0) and (-.5,0). Recall that those are the solutions that we would arrive at when we solve the equation $4x^2 - 1 = 0$. Notice also that the graph seems to have a lowest point, or minimum, at (0, -1). This point is called the vertex of the parabola. Use TRACE to see if the graph will go any lower. It doesn't, does it? The vertex is always the highest or lowest point of the parabola.

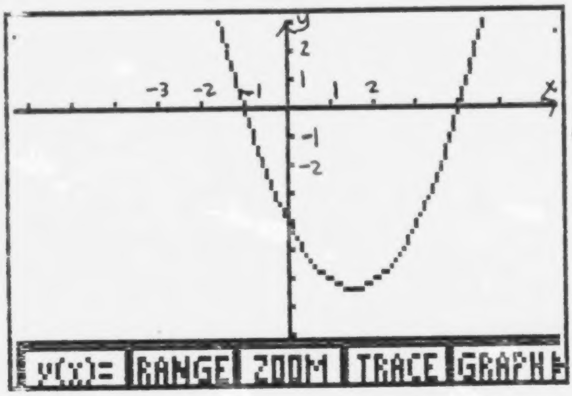
When will the graph have a highest point and when will it have a lowest point? Graph the following equations and record whether they have a U shape or a ∩ shape.

a. $y = x^2$
b. $y = -x^2$
c. $y = \frac{1}{2}x^2$
d. $y = \frac{-1}{2}x^2$
e. $y = 2x^2$
f. $-2x^2$
g. $y = x^2 + 4x + 2$
h. $y = -x^2 + 4x + 2$

What conclusion can you draw: Which part of the equation seems to affect whether the graph opens up (U shape) or opens down (∩ shape)?

Theorem: If $y = ax^2 + bx + c$ and $a > 0$, the graph will open up. If $a < 0$, then the graph will open down.

Consider the graph of the following equation and put it on your calculator in ZDECM: $y = x^2 - 3x - 4$



Notice that the graph is symmetric about some line. Which line? What point on or points on the parabola does this line pass through?

This is not a coincidence. Look back at your graphs of the parabolas from the beginning of this section. Do they all have this same symmetry property?

They should. The line of symmetry for a parabola passes through the vertex. This fact is very helpful in determining the

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coordinates of the vertex from the equation. Notice also that the x-intercepts, when there are some, are the same distance on either side of the vertex; this is because of the symmetry.

In order to examine the coordinates of the vertex, we must first recall the Quadratic Formula. Remember that this formula allows us to solve any quadratic equation. Thus it can be an aid in finding the x-intercepts of a parabola.

Theorem: If $ax^2 + bx + c = 0$ and $a \neq 0$, then

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Consider those two x values:

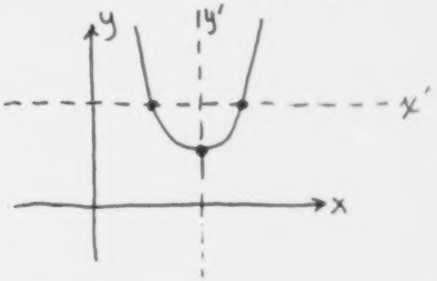
$$x = \frac{-b+\sqrt{b^2-4ac}}{2a} \quad \text{and} \quad x = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

What is their average? (Recall that an average is the number that is exactly halfway between two other numbers.) Why is finding their average important? What does it tell us?

Hopefully, when you took the average, $x = \frac{x_1 + x_2}{2}$, you got

$x = \frac{-b}{2a}$. This is the x-coordinate of the vertex since the vertex is halfway between the two x-intercepts.

Now, what if a parabola doesn't have any x-intercepts, like the sketch below? Then we can draw a new set of axes (translate the axes) so that it does cross the new axes.

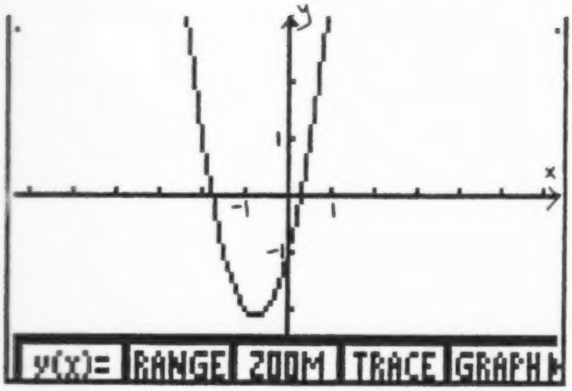


When we do that we then have some x' -intercepts and the x'-coordinate can be found in the same way. When we move the axes like that, it changes the equation, but if we were to "convert back" to the original equation, the vertex would still have the same x-coordinate.

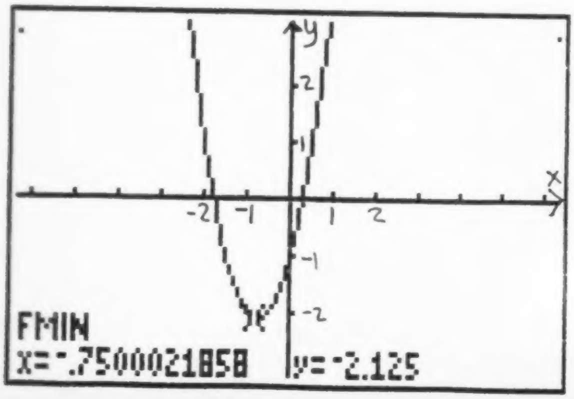
Just to make sure that this formula, $x = \frac{-b}{2a}$, works every time, graph the following equations, use TRACE (or FMIN or FMAX) to find the vertex and then compare the value you get on the graph to the value from the formula.

Example 1: $y = 2x^2 + 3x - 1$

Solution: Graph the equation in ZDECM. By using TRACE, the two "lowest values" are at the points (-.7, -2.12) and (-.8, -2.12). Because of the symmetry, we know that the vertex is halfway between those.



We can either zoom in until we get a "bottom" point, or we can use a new function on the calculator. The new button is called FMIN. We will use that one because we know we are looking for a minimum point. If you are still in TRACE, press EXIT. Then press MORE and MATH (F1). Press MORE and FMIN (F1). Move the cursor close to the low point and press enter. You should have:

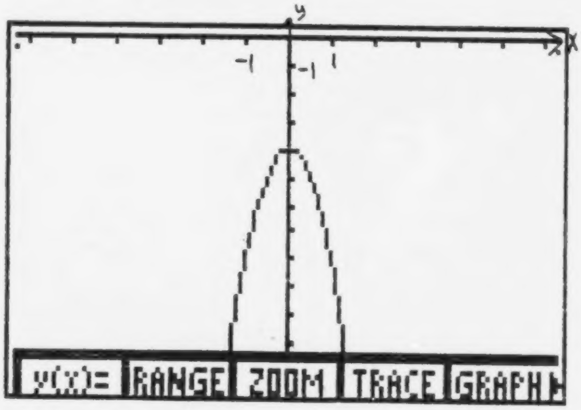


FMIN
x=⁻.7500021858 y=⁻2.125

Thus the vertex is at approximately (-.75, -2.125) from the calculator. Algebraically, we have x = $\frac{-3}{2(2)}$ = -.75 and replacing x with that value in the original equation we have $y = 2(-.75)^2 + 3\ (-.75) - 1 = -2.125$. Therefore the formula appears to work. Remember that the calculator will not always be able to give the exact answer because it does a lot of approximating in its computations.

Example 2: $y = -4x^2 - 4$

Solution: Graph this in ZDECM. Nothing showed up so examine the y-values of the equation by plugging in a couple of values for x. They will all be less than or equal to -4. So go to RANGE and adjust your window. Leave the x values alone and make the y-values : ymin = -12.4 and ymax = 0. (Notice that ymax - ymin is still a multiple of 6.2, so it is still friendly.)



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By TRACE, we find the vertex at (0, -4). We could use FMAX here in the same way that we used FMIN in example 1, but when you do so, you will find that this is the same value. By using the formula, $x = \frac{0}{2(-4)} = 0$, so that $y = -4$. Thus the formula works again.

(Notice that this parabola did not have any x-intercepts.)

Go back to the quadratic equations that you graphed on page 2 (A - F). Notice that you had to change the window on your calculator sometimes in order to get a complete graph. Find the vertices (plural of vertex) of each of those equations and record them on your paper. If you do not still have their graphs, you should probably record them now as well. Then graph the following equations: (i) $y = -(x - 3)^2 + 9$

(ii) $y = -(x + 1)^2 - 2$

(iii) $y = -\frac{3}{4}(x - 2)^2 + 3$

What do you notice about these three graphs? They are the same as the graphs in B, C, D, respectively. If you complete the square in those three equations, you will arrive at the forms in (i), (ii), and (iii), respectively. This form of the equation of a parabola is called the standard form:

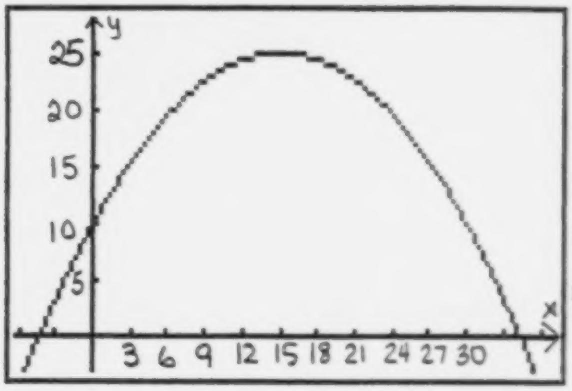
$y = a(x - h)^2 + k$,

where h is the x-coordinate of the vertex and k is the y-coordinate of the vertex.(Check this with your vertices that you have for A - F.)

When the equation of the parabola is not exactly $y = x^2$, the parabola has been transformed. The values h, k and a are transformations. The h and k affect where the vertex is and the a affects whether the parabola opens up or down and how wide or skinny it opens. (Look at the graphs of a - h).

Because of the graphing calculator, if we are given the equation, we can find the graph and the important features pretty easily. But, if we start with the graph, like the path that a thrown ball might travel, it would be helpful to be able to find the equation.

Example 3: Find the equation of the parabola:



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Solution: The parabola passes through the point (0,10) and has vertex (15,25). Thus h = 15 and k = 25. The other point will help us determine a. We know a is negative, because the parabola opens

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down. So far we have $y = a(x-15)^2 + 25$.

Plug in (0,10): $10 = a(0-15)^2 + 25$

$$-15 = 225a$$

$$-1/15 = a$$

Thus $y = -1/15(x-15)^2 + 25$. Graph this on your calculator in an appropriate window to check that the graph of this is the same as the graph above.

Example 4: Using algebraic methods, find the vertex, the maximum or minimum value, and the intercepts of the parabola given by $y = -4x^2 - 7x + 2$.

Solution: The vertex has coordinates:

$$x = \frac{-b}{2a} = \frac{-(-7)}{2(4)} = \frac{7}{8}$$

$$y = -4\left(\frac{7}{8}\right)^2 - 7\left(\frac{7}{8}\right) + 2 = -7.1875$$

Thus, the vertex is (7/8, -7.1875). And since $a = -4$ is negative, the maximum value is -7.1875.

Intercepts: Solve $-4x^2 - 7x + 2 = 0$. Use the quadratic formula:

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(-4)(2)}}{2(-4)}$$

$$x = \frac{7 \pm \sqrt{81}}{-8}$$

$$x = \frac{7+9}{-8} = \frac{16}{-8} = -2 \quad \text{or} \quad x = \frac{7-9}{-8} = \frac{-2}{-8} = \frac{1}{4}$$

The x-intercepts are (-2,0) and (1/4, 0).

Now you try some problems on your own. If you need to, go back to the example that is listed in parentheses for the problem to get help. When you finish those or when you feel like you are ready, go on to the review problems. These will test whether you understand how to use what you have learned in this section.

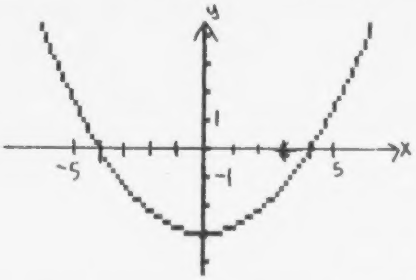
Exercises for Section 1

Graph. State the x-intercepts and the vertex. (Examples 1, 2)

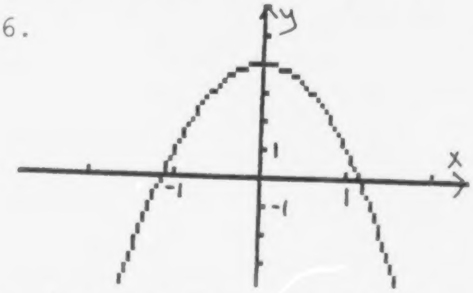
1. $y = 0.17x^2 + 2x - 3.2$
2. $y = 2x^2 - x + 4$
3. $y = x^2 - 5x + 6$
4. $y = -3x^2 + 4x - 1$

Find the equation of the parabola from the graph. (Example 3)

5.



6.



Find the vertex, the x-intercepts, and the maximum or minimum of the parabola given by the following equations using an algebraic method. (Example 4)

7. $y = 2.1x^2 - x + 1.2$
8. $y = 2x + 4x - 9$
9. $y = (x - 6)^2$
10. $y = -3x - 4x^2$

Review Problems for Section 1

1. Which of the following equations represents a parabola?

a) $y = 2x^2 - 5x + 8$

b) $y = 3x + 4.6$

c) $y = .5x^2 + 4x^3$

d) $y = x^2 - 2x + 1$

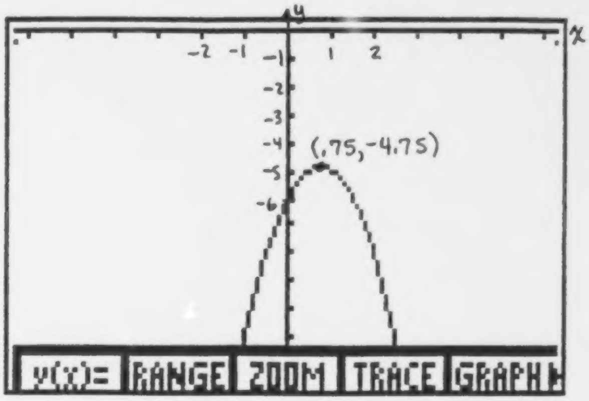
e) $y = x + 8x^4$

f) $y = (x - 3)^3 + (x - 3)^2$

g) $y = 2x^2 - 5$

h) $y = 3x + .7x^2$

2. Find the equation of the parabola:



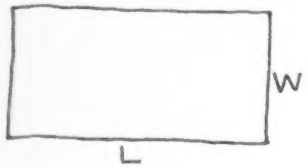
3. Find the vertex, maximum or minimum, and the intercepts of the parabola given by $y = 2x^2 - 2x - 4$.

Section 2

Sometimes quadratic equations can be used to solve application problems.

Example 1: If 100 meters of fencing will be used to fence a rectangular region, then what dimensions for the rectangle will maximize the area?

Solution: Draw a picture. We have a rectangle with a length L and a width W. The fence will go around the rectangular region, so we can relate L and W by the perimeter of the rectangle. $P = 2(L + W)$ Thus $100 = 2(L + W)$ or $L = 50 - W$.



Since the area of a rectangle is $A = LW$, we now have $A = (50 - W)W$ or $A = 50W - W^2$. Since the coefficient of W^2 is negative, we will get a maximum value. Graph the parabola and find the vertex. The W value of the vertex is 25, so the maximum area is 625 square meters.

Sometimes instead of maximizing or minimizing in a problem situation, we actually need the x- intercepts. We can find them either graphically or by using the quadratic formula.

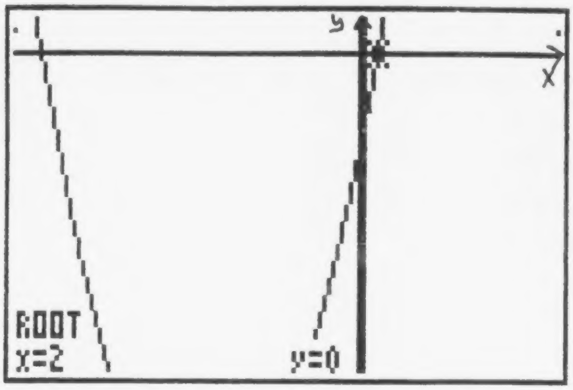
Example 2: A rectangular pool 30 feet by 40 feet has a strip of concrete of uniform width around it. If the total area of the pool and the concrete is 1496 square feet, find the width of the strip.

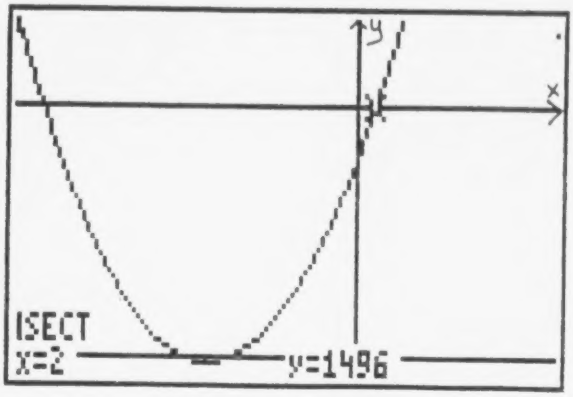
Solution: Draw a picture. Let x be the width of the strip. The pool and the concrete strip then have the dimensions shown in the picture. $A = (30 + 2x)(40 + 2x)$ which is 1496.

$$1496 = (30 + 2x)(40 + 2x)$$

This can be done by the quadratic formula, by multiplying the right hand side and subtracting 1496 from both sides to get zero on the left. Then you have $0 = 4x^2 + 140x - 296$ and the quadratic formula yields the values $x = 2$ or $x = -37$. Obviously, the width of the concrete strip cannot be -37. Hence the strip is 2 feet wide.

This problem could be solved by graphing $y = 4x^2 + 140x - 296$ and finding the x-intercepts (Do it.) This problem could also be solved by finding the points of intersection of $y1 = 1496$ and $y2 = (30 + 2x)(40 + 2x)$. Using this way, no multiplication has to take place first. (Why is that useful?)





Example 3: One pipe can fill a tank in 5 hours less than another. Together they can fill the tank in 5 hours. How long would it take each alone to fill the tank?

Solution: Let x be the time it takes Pipe 1 to fill the tank. How long does it take Pipe 2?

Now, Pipe 1 can fill the tank in x hours so we have $\frac{1 \text{ tank}}{x \text{ hours}}$.

What is the ratio for Pipe 2?

The two pipes working together fill the tank in 5 hours, so we have $\frac{1 \text{ tank}}{5 \text{ hours}}$.

Pipe 1 ratio + Pipe 2 ratio = together ratio

$$\frac{1}{x} \qquad + \qquad \frac{1}{x-5} \qquad = \qquad \frac{1}{5}$$

This does not seem to be a quadratic, but we may clear of fractions to eliminate the denominators and this will yield a quadratic equation.

The common denominator is $5x(x-5)$, so multiply each term by that expression and you will get:

$$5(x-5) + 5x = x(x-5) \quad \text{or} \quad 10x - 25 = x^2 - 5x$$

We can solve this either by graphing or by the quadratic formula like in Example 2 above. The x values will be

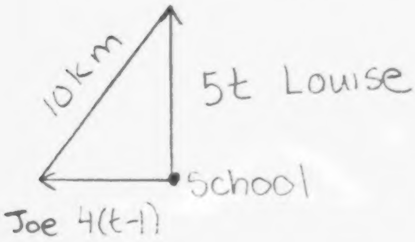
$$x = \frac{15 - \sqrt{125}}{2} \approx 1.91 \qquad \text{or } x = \frac{15 + \sqrt{125}}{2} \approx 13.09$$

If you had graphed the two sides of the equation and looked for the points of intersection, you would have arrived at the same solution. Which one is most logically the correct x value for this situation? If you said 1.91, then the other pipe was able to fill a tank in 1.91 - 5 = -3.09 hours which would be interesting! If you said 13.09, then it takes the other pipe 8.09 hours. Does it seem reasonable that the time it takes them to work together is shorter than either of their times?

Example 4: Joe and Louise attend the same school in the Mojave Desert. Both students are members of the cross-country team, and their afternoon workout entails running to their homes after school. Louise leaves first, running due north at a steady pace of 5 kilometers per hour. Joe spends an hour stretching first and then leaves an hour after Louise started. Joe heads due west and runs at a steady pace of 4 kilometers per hour. If Louise left at 3:00p.m., at what time will they be precisely 10 kilometers apart?
Solution: Draw a picture. This will have a triangle shape and since north and west are perpendicular to each other, we will have a right triangle.

Let t = Louise time. Then $t - 1$ = Joe's time (Why?)

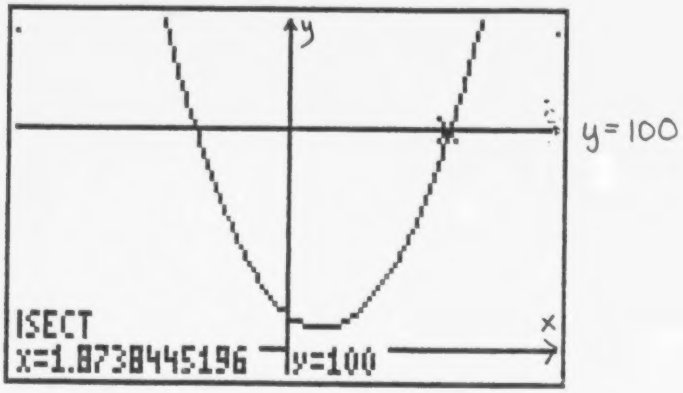
Thus Louise will have gone $5t$ and Joe will have run $4(t - 1)$ when they are 10 miles apart.



Using the Pythagorean Theorem (since we have a right triangle):

$$10^2 = (5t)^2 + (4(t - 1))^2$$

Graph both sides of the equation and find the intersection points.



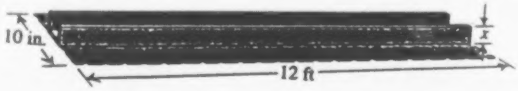
Since Louise's time cannot be negative, the t value must be approximately 1.87 hours. In hours and minutes what is this? (Recall .87 means .87 of an hour.) What time did this make?

Now you try some problems on your own. If you need to, go back to the example that is listed in parentheses for the problem to get help. When you finish those or when you feel like you are ready, go on to the review problems. These will test whether you understand how to use what you have learned in this section.

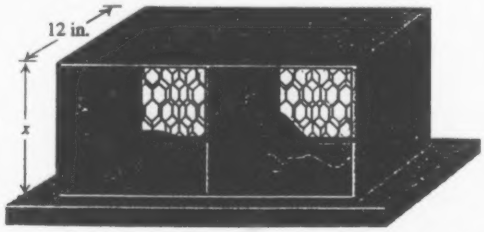
Exercises for Section 2

1. Mona Kalini gives a walking tour of Honolulu to one person for \$49. To increase her business, she advertised at the National Orthodontist Convention that she would lower the price \$1 per person for each additional person, up to \$49 people. Write the equation that represents her revenue in terms of the number of people on the tour. What number of people will maximize her revenue? What is the maximum revenue for her tour? (Example 1)

2. Seth has a piece of aluminum that is 10 inches wide and 12 feet long. He plans to form a rain gutter with a rectangular cross section and an open top by folding up the sides as shown in the picture below. What dimensions of the gutter would maximize the amount of water that it can hold? (Example 1)

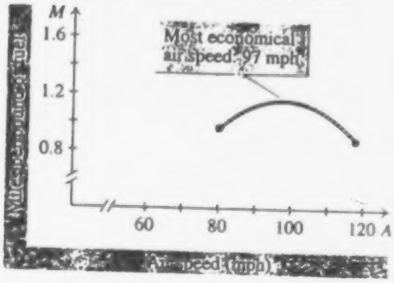


3. Sharon has a 12-ft board that is 12 inches wide. She wants to cut it into five pieces to make a cage for two pigeons, as shown below. The front and the back will be covered with chicken wire. What should be the dimensions of the cage to minimize the volume and use all of the 12-ft board? (Example 1)



4. Lindbergh and Hall used a graph similar to the one below to determine the air speed at which the number of miles per pound of fuel, M, would be a maximum for a fully loaded plane. The curve shown here was determined by testing the loaded plane at Camp Kearney in 1927. Using empirical data (data acquired by experiment), Lindbergh figured that the most economical airspeed would occur at the highest point on the curve, at roughly 97 mph. In the picture below, the curve appears to be a parabola. I we assume that it is a parabola, then M is a quadratic and its equation is approximately $M = -0.000653A^2 + 0.127A - 5.01$ where A is the air speed. What value of A would maximize M? (Example 1)

Miles per Pound of Fuel at Takeoff

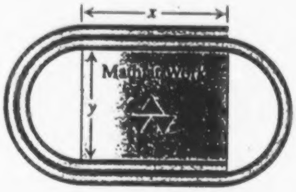


5. The height of an object thrown upward with an initial velocity of 64 feet per second after t seconds is given by $h = 64t - 16t^2$. When will the ball hit the ground, assuming that it was thrown up at time $t = 0$? (Example 2)

6. Jose Canseco hits a towering fly toward the left-field seats. The height of the ball above ground level is given by $h = -16t^2 + 192t$, where h is the height of the ball in feet and t is the number of seconds that have passed since the ball left Canseco's bat. How long will the ball be in the air before it hits the ground in the outfield? (Example 2)
7. The bathtub faucet can fill the bathtub to the rim in 10 minutes less than it takes the water to totally drain out of the tub. If it takes 15 minutes for the tub to get filled while the plug is out and the water is running, how long does it take a full bath tub to drain alone? (Example 3)
8. Kara's rescue team is practicing a maneuver that one day might save the life of a drowning person. Kara, dressed in a lifejacket, is lowered into the current of the Trinity River by means of a rope attached to a harness that she wears. The current takes her downstream. The other end of the rope is anchored to the edge of the dock, which is 10 feet above the level of the river. If 50 feet of line is played out, how far downstream is Kara? (Example 4)

Review Problems for Section2

1. Solve $y^2 = \frac{3}{2}(y + 1)$ using the quadratic formula.
2. Solve $x^2 - 6x = -2$ graphically.
3. Sylvia is developing a paper clip that will carry a company log in the rectangular center section as shown in the picture below. If the clip is to be made out of an 8 inch long piece of wire, then what dimensions for x and y will maximize the area of the rectangular center region? To simplify the problem, assume that the radii of the three semicircles on the ends are equal to y/2.



Section 3

Inequalities can contain quadratic expressions. These are most easily solve by graphing the equation and finding which x values satisfy the inequality.

Example 1: A company manufactures and sell flashlights. For a particular model, the marketing research and financial departments estimate that at a price of \$p per unit, the weekly cost C and revenue R (in thousands of dollars) will be given by the equations:

$C = 7 - p$

$R = 5p - p^2$

Find the prices for which the company will realize

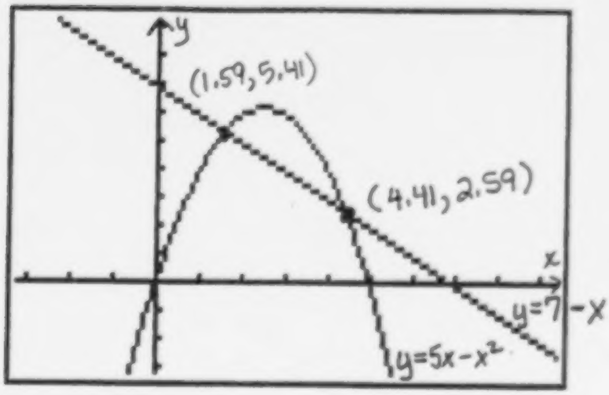
a) a profit b) a loss.

Solution: A profit will occur whenever $R > C$. The **breakeven points** are when $R = C$. Graph both equations, letting p be x on the calculator. What are the breakeven values? Use ISECT to get pretty good approximations. Where is $R > C$? (Recall that $R > C$ when the graph of R is above the graph of C.) Write the inequality for this. Where is $R < C$? What does it mean if R is less than C? Write the inequalities for this. Are there any other limitations on x? If so what are they?

Hopefully, you got the following picture. The break-even points are labeled on the graph.

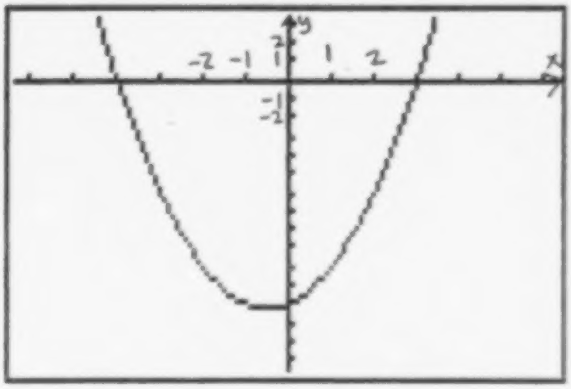
A profit will occur when $R > C$ which is when $\$1.59 < p < \$4.41$.

A loss will occur when $R < C$ which is when $\$0 \leq p < \$1.59$ or $\$4.41 < p$



Example 2: Solve the inequality $x^2 + x < 12$.

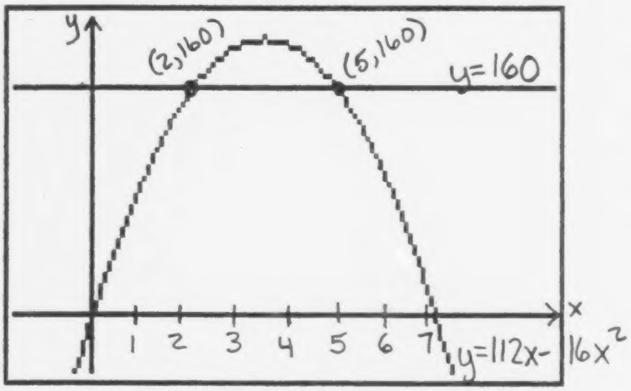
Solution: Graph the equation $y = x^2 + x - 12$ and find the x-intercepts. Remember that the problem is requesting the solution to the inequality $x^2 + x - 12 < 0$, so we want to know where the y values are negative. Looking at the graph, where does that occur?



The solution is the interval (-4, 3).

Example 3: If an object is shot straight up from the ground with an initial velocity of 112 feet per second, its distance d (in feet) above the ground at the end of t seconds(neglecting air resistance) is given by $d = 112t - 16t^2$ Find the interval of time for which the object is 160 feet above the ground or higher.

Solution: The easiest way to see this is to graph the equation $y = 112x - 16x^2$ and $y = 160$ and see where the parabola is on or above the line.



Now you try some problems on your own. If you need to, go back to the example that is listed in parentheses for the problem to get help.

Exercises for Section 3

1. A company manufactures and sell computer ribbons. For a particular ribbon, if the price is estimated at \$p per unit, the weekly cost C and revenue R (in thousands of dollars) will be given by the equations $C = 13 - p$ and $R = 7p - p^2$. Find the prices for which the company will have a profit and for which the company will have a loss. (Example 1)

2. Solve the inequality $x^2 + 5x \leq 2$. (Example 2)

3. In Example 3, find the interval of time for which the object is above the ground.

4. It is of considerable importance to know the shortest distance d (in feet) in which a car can be stopped, including reaction time of the driver, at various speeds v (in mph). Safety research has produced the formula $d - 0.044v^2 + 1.1v$ for a given car. At what speeds will it take the car more than 330 feet to stop? (Example 3)

5. In # 4, at what speeds will it take a car less than 220 feet to stop? (Example 3)

FUNCTIONS
APPLIED ALGEBRA CURRICULUM MODULE

Performance Objective: ***Students will be able to write algebraic representations of functions, to read numerical or tabular representations of functions, and interpret the meanings of functions from graphical representations.***

Direct (Linear) Functions

Investigation/Demonstration:
Jay works part-time for Mid-Town Freight Company and earns $6.50 per hour. Complete the following table to show the amount of money Jay earns (E) as a function of the number of hours (H) that he works.

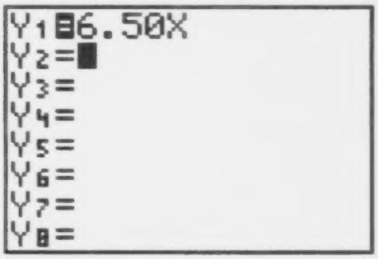
Hours Worked (H)	0	1	2	3	4	5	6	7	8	9
Earnings (E)	$0	$6.50								

Write an equation for the linear function, using H for *hours worked* and E for money *earned*. Remember that money earned depends on the hours worked.

Predict Jay's earnings after working 10 hours.   After 15 hours.   After 28 hours.

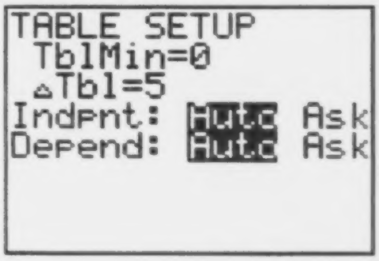
Demonstrate the table-building feature of the graphing calculator to show Jay's earnings for hours worked from 0 to 40 hours.

Press the **Y=** key and enter your algebraic representation:



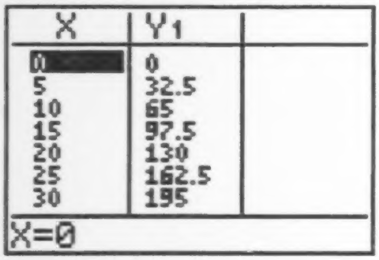
```
Y1=6.50X
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
Y8=
```

Press **2nd TblSet** (above the **WINDOW** key) and set the table:



```
TABLE SETUP
 TblMin=0
 ΔTbl=5
Indpnt: Auto Ask
Depend: Auto Ask
```

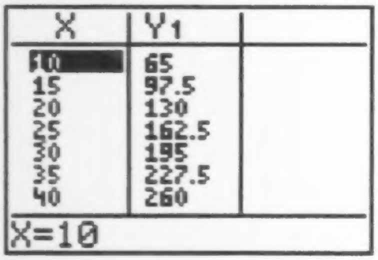
Press the **2nd TABLE** (above the **GRAPH** key) to get the first view of the table:



X	Y1
0	0
5	32.5
10	65
15	97.5
20	130
25	162.5
30	195

X=0

Press the down cursor arrow to get a further view of the table:



X	Y1
10	65
15	97.5
20	130
25	162.5
30	195
35	227.5
40	260

X=10

Activity

The amount of money that is paid to fill your automobile's fuel tank is a function of the number of gallons of gasoline. Anna chooses to fill her tank with super-unleaded gasoline priced at $\$1.28\frac{9}{10}$.

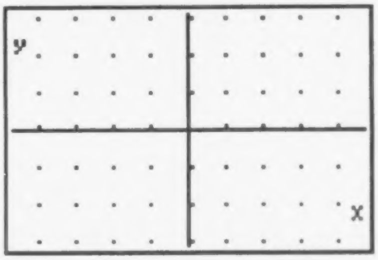
- Name two variables which might be used to describe this function.
- What does each variable represent?
- Write an equation for the linear function, using the variables that you selected.
- Construct a table showing the amount of money Anna pays for 0, 5, 10, 15, 20, 25, and 30 gallons of gasoline.
- Predict how much Anna will have to pay if she completely fills her tank with 22 gallons.

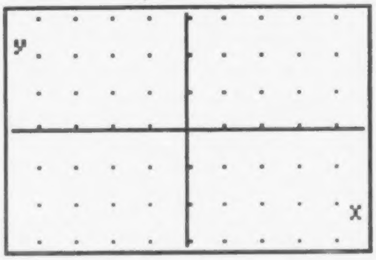
Inverse Functions

Investigation/Demonstration:

Recall that a function is **one-to-one** if *every* horizontal line intersects the graph of the function at most once.

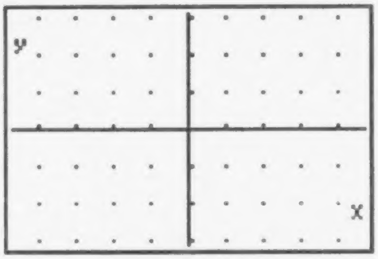
Sketch the graph of the function $y = x^2$ in the window on the left below:

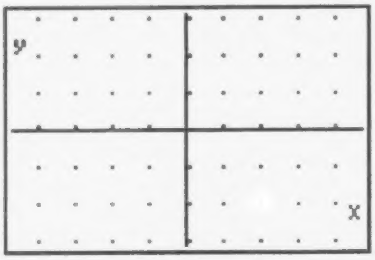




Sketch the graph of the inverse of the function $y = x^2$ in the window on the right above

Now, sketch the graph of the function $y = -x^3$ in the window on the left below:



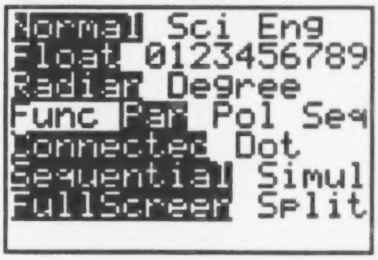


Sketch the graph of the inverse of the function $y = -x^3$ in the window on the right above.

- Which of the functions are one-to-one? How do you know?
- What features of the graphs help us to determine whether the functions are one to one?

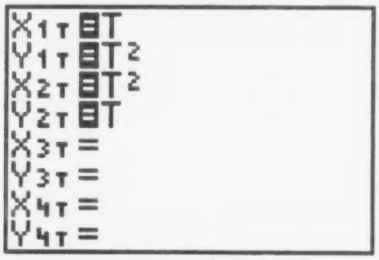
Demonstrate the accuracy of your answers by displaying the graphs of each function and its inverse using the parametric graphing mode on your graphing calculator.

Set the calculator to parametric mode by pressing the **MODE** key and then moving the cursor to highlight **Par** as shown:



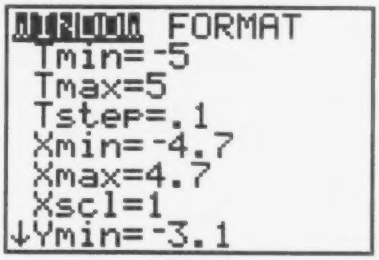
```
Normal Sci Eng
Float 0123456789
Radian Degree
Func Par Pol Seq
Connected Dot
Sequential Simul
FullScreen Split
```

Press the **Y=** key and enter the first function and its inverse as shown:



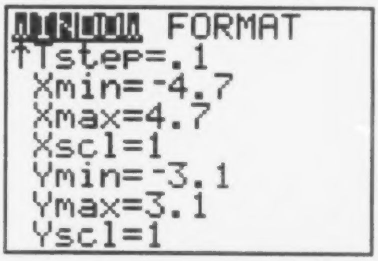
```
X1T=T
Y1T=T²
X2T=T²
Y2T=T
X3T=
Y3T=
X4T=
Y4T=
```

Set an appropriate viewing window. Press WINDOW key and use the example shown:



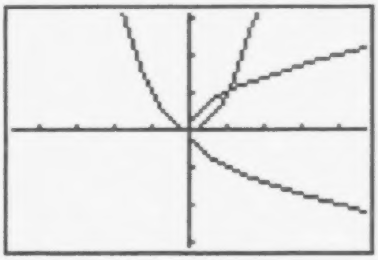
```
WINDOW FORMAT
 Tmin=-5
 Tmax=5
 Tstep=.1
 Xmin=-4.7
 Xmax=4.7
 Xscl=1
↓Ymin=-3.1
```

The continuation of the window is given here:

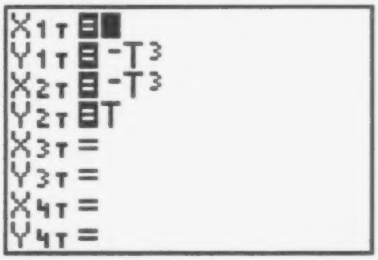


```
WINDOW FORMAT
↑Tstep=.1
 Xmin=-4.7
 Xmax=4.7
 Xscl=1
 Ymin=-3.1
 Ymax=3.1
 Yscl=1
```

Press the **GRAPH** key to see the complete graph of the function and its inverse:

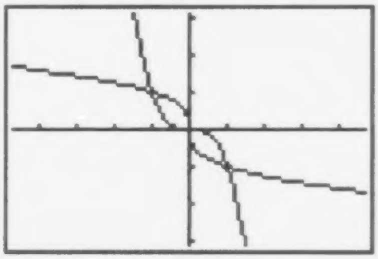


Press the **Y=** key and enter the second function and its inverse:



```
X1T=
Y1T=-T³
X2T=-T³
Y2T=T
X3T=
Y3T=
X4T=
Y4T=
```

Press the **GRAPH** key to see the complete graph of the function and its inverse:



Activity

Use a graphing calculator to determine whether each function is one-to-one:

(a) $y = x^3 + x$

(b) $y = 2x - x^3$

(c) $y = -3\sqrt{x}$

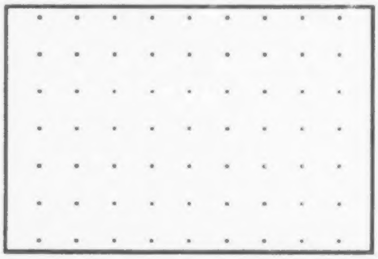
Polynomial Functions

Investigation/Demonstration:
The economy functions in such a way that we can conclude that supply curves are usually increasing (as the price increase, sellers increase production) and the demand curves are usually decreasing (as the price increases, consumers buy less). Suppose that a certain single-commodity market situation is driven by the following system:

Supply: $P = 20 + 0.2x^2$

Demand: $P = 985 - 0.1x^2$

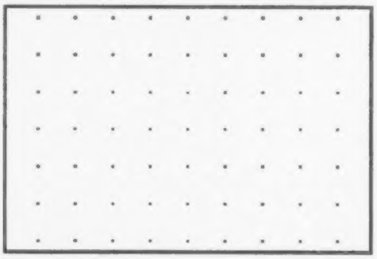
Find the graph of both the supply and the demand equations in the *first* quadrant. Let x be the number of units produced and P is the price. Sketch the graph in the window below. Be sure to specify the scale factors that you used to find the graph.



Determine the equilibrium price. The equilibrium price is the price at which supply is equal to demand.

Activity
Given a sheet of thin metal with dimensions 25 inches by 30 inches, squares of equal sides are to be cut from the corners of the sheet. The sides are then to be turned up in such a way as to form a box with no top. Let x be the side length of the squares that are cut out.

- Draw a diagram of the problem situation
- Express the volume V(x) of the box as a function of x.
- Find a complete graph of the problem situation and sketch in the window:



- What are the dimensions of the box that will produce maximum volume?
- For what values of x will volume of the box always be at lease 1000 cubic inches.

Exponential Functions

Investigation/Demonstration:

Suppose Joanna invests $1000 at 8.5% interest compound annually. How long will it take to double her investment?

Using a TI-82 graphing calculator, follow these simple steps. At the HOME screen enter the expression **1000+(1+.085)** and press the **ENTER** key. The display should show 1085. Now enter the expression **Ans(1+.085)** and press the **ENTER** key. Each time the **ENTER** key is pressed, one interest period has been calculated and the balance in the account at the end of that period is shown. Successive presses of the **ENTER** key represents, in this case, another year of compounded interest added to the original balance. Be sure to count the number of key presses of the **ENTER** key so that an accurate count of the years is a result. When the account reaches or exceeds 2000, the account has tripled in value and the number of **ENTER** key presses is the answer to the question. Test your answer by entering **1000+(1+.085)^"your answer"** and press the **ENTER** key.

The Rule of 72 is a way for approximating the number of periods that it will take for an investment to double. The conceptual formula is given as:

$$\text{Number of periods} = \frac{72}{\text{Interest rate per period (without the \% sign)}}$$

Using the problem above, the number of periods needed for $1000 to double with an interest rate of 8.5% would be calculated as 8.47 years. How does this approximation compare with the result that was obtained previously?

Activity

Luann graduates from technical school at age 23 and begins making monthly deposits of $50 into a retirement account that pays 8.25% compounded monthly. Use the following formula:

$$\text{Future Value of an Ordinary Annuity: } S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

where S = future value of an ordinary annuity consisting of n equal payment
R = amount of each equal payment
i = interest rate per pay period (payment interval)

- Luann plans to make $50 deposits in her retirement account each month until she retires at age 62. What will be the value of her retirement fund when she retires if the interest rate remains constant at 8.25%?
- Find a complete graph for S as a function of n years for an interest rate of 8.25% and monthly deposits of $50.

- Compare the amount in the retirement account upon Luann's retirement if the interest rate was 11.75% for the life of the account.
- What rate of interest would be necessary in order for Luann to accumulate $1 million dollars upon her retirement?

Blair plans to buy a car and desires to finance it with the her company's credit union. The credit union will finance $13,000 for 48 months at an annual percentage rate (APR) of 10.5% so that Blair can purchase the car. Use the following formula:

Amount of Mortgage Payment(Present Value): $A = P\left[\frac{i}{1-(1+i)^{-n}}\right]$

where A = amount of each equal payment
P = amount financed
i = interest rate per pay period (that is, $i = \frac{APR}{12}$ if the periods are months)

- Find Blair's monthly payment for each of 48 monthly payments at 10.5%
- What is the monthly payment if she finances the car for 36 months?
- Compare the total amount paid (that is, the total of the payments) for each of the two payment periods--48 months and 36 months.
- What would be Blair's monthly payment if she paid 60 monthly payments at 7.5%?

Rational Functions

Investigation/Demonstration:

Jaime rode his motorbike 18 miles to school and then completed his trip by bus. The entire distance traveled was 110 miles. The average rate of the bus was 15 miles per hour faster than the average rate of motorbike.

Find an algebraic representation that gives the total time t required to complete the trip as a function of the rate x of the bus.

Suppose Jaime has 2 hours to complete the 110-mile trip. Use a graphing calculator to find the rate of the bus. Use an algebraic method to confirm your answer.

At what possible rates of speed must Jaime travel by bus to ensure that the total time for him to complete the trip is less than 2.5 hours?

Activity

The concentration of pure solute in a solution is given by the conceptual formula:

$$\text{Concentration of solute} = \frac{\text{Quantity of solute}}{\text{Total quantity of solution}}$$

A quantity of solution that is 80% barium is to be added to 5.2 liters of a 48% barium solution to produce a solution that is at least 65% barium?

- Write the algebraic representation of the concentration $C(x)$ as a function of the quantity x of pure barium.
- Find a complete graph of the algebraic representation and sketch below

- What part of the graph represents the problem situation?
- What is the solution to the problem?

Radical Functions

Investigation/Demonstration:

The surface area of a right circular cone, excluding the base, is given by the formula:

$S = \pi r\sqrt{r^2 + h^2}$ where r is the radius and h is the height.

The volume of the cone is $V = \frac{1}{3}\pi r^2 h$

Suppose the height of the cone is 18 feet. Find the algebraic representation and a complete graph of the surface area S as a function of the radius r.

If the height of the cone is 21 feet, what radius produces a surface area of 150 square feet?

Suppose the volume is 350 cubic feet. Find an algebraic representation and a complete graph of S as a function of r.

Find the dimensions of a cone with volume 350 cubic feet that has the minimum surface area.

Activity

Televisions screens are rectangular with the measure of the diagonal given as the size of the television. Consider a television that has a 10" diagonal screen.

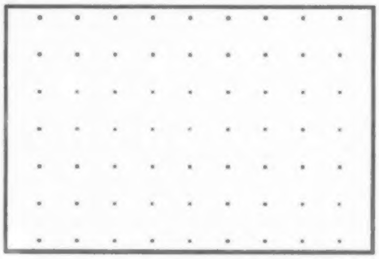
- Find the algebraic representation for the length L of the screen as a function of the width x.
- Find a complete graph of the problem situation.
- List the dimensions for *three* different rectangular screens that have a 10-inch diagonal.

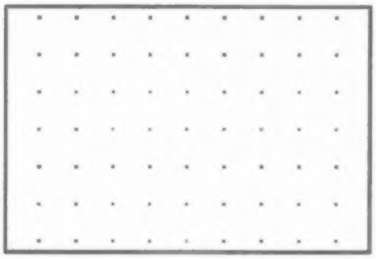
Absolute Value

Investigation/Demonstration:
Determine the *domain* and *range* for the absolute value functions:

(a) $g(x) = |x+5|$ (b) $f(x) = \left|\frac{x}{x-4}\right|$

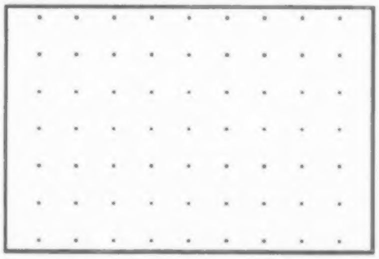
Find a complete graph for each of the functions given above and sketch them in the windows given below.





Activity

Find a complete graph for the absolute value function $f(x) = 2 + |x-3|$ and sketch in the window given.



- What is the domain for f(x)?
- What is the range for f(x)?

Proportionality Constants

Investigation/Demonstration:
On centimeter grid paper construct perfect squares that measure 2 cm, 4 cm, 6 cm, 7 cm, 9 cm, 10 cm, and 11 cm on the same sheet with no overlapping. Measure the diagonal of each perfect square to the nearest 0.1 centimeter and record your measurements in the table. Calculate the ratio of the *diagonal length* to the *side length* and record these ratios to three decimal places in the third column of the table.

Side Length	Diagonal Length	Diagonal/Side
2 cm		
4 cm		
6 cm		
7 cm		
9 cm		
10 cm		
11 cm		

What do you observe about the ratios in the third column above? What is a reasonable value for this ratio?

The ratio found in this demonstration is called a *constant of proportionality*. Explain how this constant may by used to find the diagonal length for a square that has a side measure of 50 centimeters.
What is the diagonal length for a square that measures 50 cm on a side?

Activity
Use a tape measure and select *six* circular objects in your room or outside your room to find the measures of the circumference and the diameter. Record your measurements in the table and then calculate the ratio of the *Circumference (C)* to the *diameter (d)* and record in your table.

Circumference	Diameter Length	Circumference/Diameter

- What do you observe about the ratios in the third column above?
- What is a reasonable value for this ratio?
- Did the selection of the unit of measure affect your results?
- Calculate the *circumference* that measures 50 cm on its diagonal.
- Calculate the *diameter* of a circle whose circumference is 120 cm.

Curve-sketching

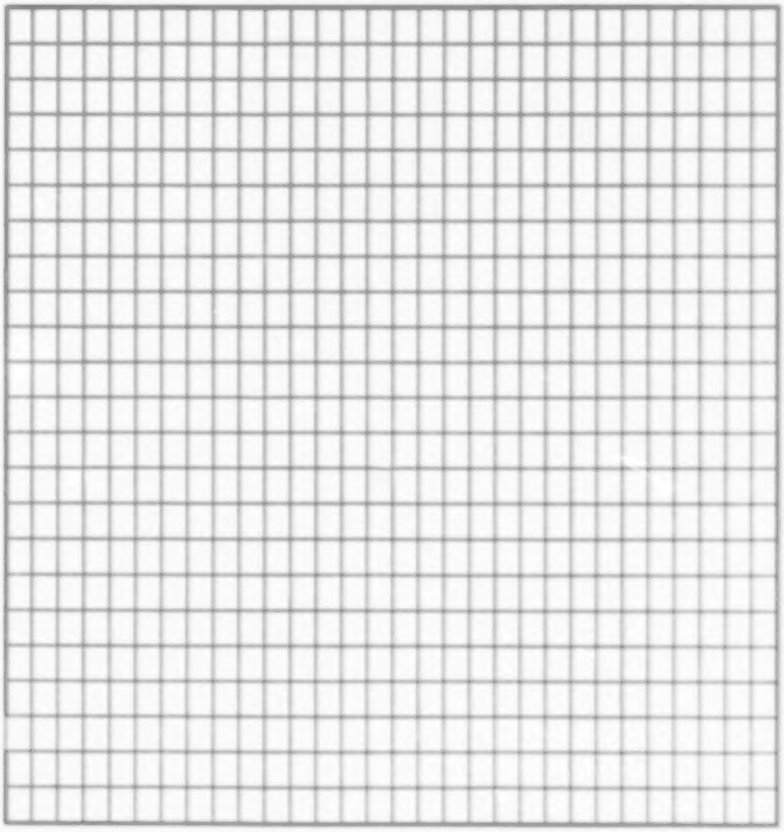
Investigation/Demonstration:

Consider the table of values for x and y given for the function $y = x^2$

X	0	1	2	3	4	5
Y	0	1	4	9	16	25

Construct the graph of the ordered pairs on the grid by following these steps:

(1) Draw an x-axis and a y-axis.
(2) Locate the ordered pair (0,0)
(3) Locate the ordered pair (1,1) by lightly drawing a segment to the right 1 unit, then up one unit from the point (0,0).
(4) Continue locating the other points by drawing a segment right 1 unit, then up 3 units, 5 units, 7 units, 9 units, etc.
(5) Connect the points with a smooth curve.
(6) Use reflection symmetry across the y-axis for each of the points in Quadrant I.
(7) Connect the points in Quadrant II to complete the parabola.



Activity

Complete the table of y-values for the function $y = \frac{1}{2}x^2$

X	0	1	2	3	4	5
Y	0					

Construct the graph of the ordered pairs on the grid below using similar steps to those in the demonstration above.

Evaluation Instrument

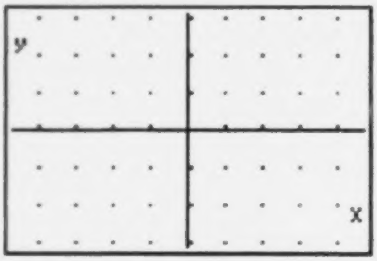
1. Write an equation for the linear function, using H for *hours worked* and E for money *earned*, given that the rate is $9.75 per hour.

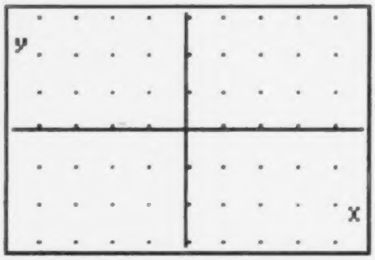
2. Build a table for problem #1 showing *hours worked* and *money earned* for 0 to 40 hours in 5-hour increments.

3. Is the function $y = x^4 + 2$ one-to-one?

4. Is the function $y = 2x + x^3$ one-to-one?

5. Sketch the graph of the inverse of the function given in #3 in the window on the right below.



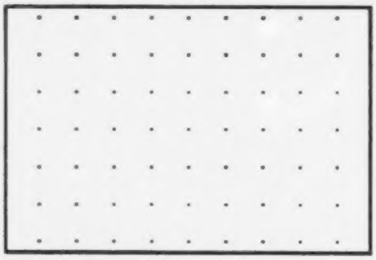


6. Sketch the graph of the inverse of the function given in #4 in the window on the right above.

For questions 7 - 9, use this problem situation: A flower garden that is 25 feet wide by 30 feet long is completely surrounded by a sidewalk of uniform width.

7. Write the algebraic representation for the total area of the garden, including the sidewalk area.

8. Find a complete graph of the algebraic representation given in #7.



9. What is the width of the sidewalk if the total area of the garden, including the sidewalk is 900 square feet.

10. Use the Rule of 72 to determine how long it will take an investment of $1000 to double at 8.5% interest compound annually.

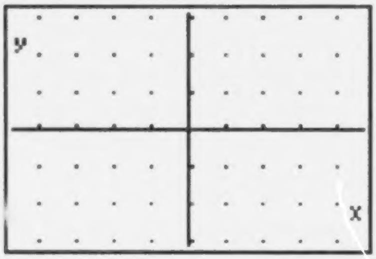
11. Blake plans to buy a car and finance it at his local bank. The bank will finance the purchase price of $10,000 for 36 months at an annual percentage rate (APR) of 12.5%. What will be the monthly payment?

12. What will be the total pay back for the car in #11?

13. Consider a television that has a 20" diagonal screen. Give the dimensions for a *square* screen that has a 20-inch diagonal.

14. Determine the *domain* and *range* for the function $g(x) = 2|x - 3|$.

15. Find a complete graph for the function given in #14 and sketch the graph in the window below.



Materials required for teaching this unit:

TI-82 Graphing Calculators
TI-82 Overhead Viewscreen
Centimeter rulers
Tape Measurers
Centimeter grid paper
Quadrille paper
Cylindrical objects

USE OF STATISTICS

APPLIED ALGEBRA CURRICULUM MODULE

COURSE OBJECTIVES are detailed as an attachment to this outline. The instructor should become familiar with these objectives before proceeding. The objectives may be shared with the students as the instructor deems necessary, or may be distributed to the students as a single document.

INTRODUCTION:

This part of the algebra course is designed to introduce the students to basic concepts of statistical process control (SPC) and meet the attached objectives. This will be accomplished by furnishing the students with a typical workplace scenario accompanied by a set of normal data representing the measured results of the process described in the scenario. Throughout the course of the lesson plan, this data will be analyzed using concepts and definitions introduced by the instructor. The students will learn how to use a statistical calculator to determine the mean and standard deviation of the data. These statistics will then be used to develop control limits for the process. The lesson plan is presented in outline form for the purpose of simplicity and ease of referral.

MATERIALS NEEDED:

- Statistical Calculator (one per person in class)
- Vernier Calipers
- M&M Candies
- Handouts (in order of appearance)
  - Tabulated Process Data from Scenario
  - Causes of Variation Exercise
  - Ordered Data from Scenario
  - Counting Rules
  - Probability Distributions
  - Areas Under the Normal Curve
  - Control Charts
  - Control Charts Exercise

LESSON PLAN

I. Present the class with the following **SCENARIO**: Workers at a manufacturing facility assembling portable video games were interested in improving their assembly process. Dated equipment and machinery were replaced, suspected problems investigated, and an intensive training effort undertaken. Six months later, the time required to produce a single unit from start to finish was measured and the results tabulated in minutes.

   A. Pass out the tabulated data to the class and ask them to look over it. Inform them that plant management was pleased to learn that the average amount of time to produce a unit was found to be LESS than before.

   B. Ask the following questions:

      1. Does the data indicate that there are still problems with the process?

      2. If there is a problem, how could we know?

   C. Regardless of the answers offered by the students, inform them that the data is actually meaningless unless analyzed using statistical concepts. The answers lie in the data itself, we just have to figure out how to extract them.

II. **PROCESSES**. Lead the class in a discussion of what constitutes a PROCESS. This term should not be foreign to most students. While a concise definition would be hard to arrive at, when we talk about industrial processes, we mean **any and all steps or activities necessary to achieve some end**. That end may be a final product or a service rendered. In the case of a laboratory, it may be a final result of an analysis. Processes take place all around us at all times. Most of these processes are measurable. Most of the problems that occur within processes are measurable. Statistics help us to make these measurements and improve our processes.

A. **Define VARIATION as the differences between items from the same process**. Variation is the opposite of consistency. On an industrial level, **PROCESS IMPROVEMENT is defined as reducing or eliminating process variation**. One of first steps in identifying variation is to examine the possible causes of variation in a process. These causes fall into two (2) general categories:

   1. **EXPECTED causes of variation - random, non-specific causes of variation** that we can usually do nothing about unless we replace part or all of the process. This type of variation is very difficult to pinpoint, and has very LITTLE impact on the process itself.
   2. **ASSIGNABLE causes of variation - specific reasons why the process has changed**. Assignable causes of variation can often be identified AND corrected. These causes make BIG differences in the process.

B. Pass out the CAUSES OF VARIATION exercise to the class. This exercise consists of a list of possible causes of variation in monthly electric bills. The students should identify which causes would be EXPECTED and which would be ASSIGNABLE by marking an E or A in the space beside each cause. Allow the class time to complete the exercise on their own, then work through the exercise together, allowing those who desire to defend their choices. This activity should only take a few minutes.

C. ACCURACY and PRECISION are statistical terms related to the measurement of variation in a process. Present the class with the following definitions:

   1. **PRECISION - the degree to which repeated measurements of the same unit result in the same value.** Many analytical procedures have a stated "precision". Repeatability is considered to be a statistical measure of precision. The only

way to INCREASE the precision of a procedure is to DECREASE the variation.

2. **ACCURACY - the closeness of agreement between an observed value and the "correct" value.** Many analytical procedures also have a stated "accuracy". Accuracy and precision have very little to do with each other. Some processes have good accuracy, but poor precision, or vice versa. Some have good accuracy and good precision or vice versa. However, some assignable causes of variation would affect BOTH the accuracy and precision.

III. STATISTICS. Since we have established that process variation is measured in statistical terms, we should now introduce other statistical terms. These terms will help us to further understand how statistics can be used to determine whether or not we have a problem with our process.

A. **POPULATION - the total collection of units from a common source.** If we were doing a study to determine the political preferences of teenagers in the United States, then "teenagers in the United States" would be our population. If, instead, we were doing a study to determine how many teenage Republicans in the United States favored gun control, then our population would be "teenage Republicans in the United States". Our population, then, is whatever we conceive it to be, limited only by what we are trying to measure. For this reason, when we measure the time taken to produce a single unit at the video game factory, the process itself becomes our population, or more specifically, the total collection of times for producing EACH unit.

B. **SAMPLE - a subset of units collected from the population.** We cannot measure the population unless we first sample it. SAMPLING refers to looking at PART of something in order to make a decision about ALL of it. The "part" is our sample, and "all" is our population. One of the most common examples of a sampling device is the thermostat on the wall. Basically, we set a desired temperature. The thermostat "samples" the temperature of the air in the immediate vicinity and uses that to decide if more heating or cooling is needed for the whole room. There is no such thing as a thermostat that measures the temperature of ALL the air in the room before it acts. When we measure the time it takes to produce a single unit at the video game factory, we are sampling the population. The thermostat can make a decision after only one sample, but we will need several samples to make decisions about our process. This is simply because our process has the potential for greater variation.

C. **MEAN - a statistical measure of the average of the data.** Each data point represents the result of a sample. Every time we sample, we will have variation. If we add all of the data points together and divide by the number of data points we have, we will have calculated the **SAMPLE MEAN.**

1. Have the class use calculators to determine the **SAMPLE MEAN** of the data presented in the scenario. Since all of the data points are reported to the nearest tenth of a minute, they should round their answer to the nearest tenth. The result should be 25.0 minutes.
2. We can now say that "based on our data" it takes an average of 25.0 minutes to produce a single unit from start to finish. Point out that MORE or FEWER samples COULD have resulted in a different sample mean. Had we the capability (and time) to measure the time taken to produce EVERY unit, we could calculate the actual **POPULATION MEAN**. Since the latter option is not practical, industrial facilities generally use the sample mean to ESTIMATE the population mean. **The number of samples we have is called our SAMPLE SIZE, and is designated using a lower case "n"**. As **n** increases, the

sample mean becomes a BETTER estimate of the population mean.

3. Numerical descriptors of the SAMPLE, such as the mean, are called **STATISTICS**. Numerical descriptors of the POPULATION, such as the mean, are called **PARAMETERS. Sample statistics are used to draw conclusions about population parameters**.

D. **MEDIAN - the "middle" observation of the ORDERED data.** Sometimes knowing the average or mean of the data is not enough. We can used the median to see how the data is distributed. The median represents the "50% point" of the data. Half the data points will take on the value of the median OR LESS, and half will take on this value OR MORE. Hand out the ordered data from our scenario to the class. The median is determined by first placing the data in ascending (or decending) order. If **n** is odd, the median is the "middle" data point. For example, if **n** = 21, then there are 10 data points BELOW the 11th data point, and 10 data points ABOVE the 11th, so the 11th data point is our median. If **n** is even, we have to average the two center data points to determine the median. In our data, **n** = 30, so we have to average the 15th and 16th data points. Have the class do this. The result is 24.85 minutes, which we round to 24.8 minutes. We can conclude that half the units sampled took 24.8 minutes or less to produce, and half took 24.8 minutes or more. Note that this is very close, but not equal to the mean.

E. **MODE - the value which occurs most often in the data.** Note that there MAY be more than one mode in a data set. If the two most frequently occuring values occur the same number of times, we say the set is BIMODAL, and report both values. The mode is more easily determined when the data is ordered, simply because like values are listed together. Have the class determine the mode of the data presented to them. The result should be 24.7 minutes. Very soon we will be looking at how to use the mean, median, and mode to draw conclusions about our process, but first we must discuss probability.

F. **PROBABILITY - the ratio of the number of times an event occurs to the number of chances it has to occur.** The key to probability is to remember that it predicts what will happen IN THE LONG RUN. For example, the mode of our data set was 24.7 minutes. We could calculate the probability of the next part produced taking exactly 24.7 minutes. We had 30 data points, and 24.7 occured 4 times. 4/30 = 0.133, resulting in a 13.3% probability of this event occuring again. How good is this probability? That is very dependent upon the number of samples we have. Obviously, if we had 100 samples, we would have a better estimation of the actual probability. Let's do more examples.

1. If we roll a fair die, what is the probability of getting a 4? The probability of getting ANY number on a face is the same - 1 out of 6, or 0.1666. Note that probability is often reported as a decimal fraction instead of a percentage.
2. Rolling the same die, what would be the probability of getting a 2 OR a 5? **Anytime we use OR to distinguish between two separate events, we must ADD the probabilities of the separate events together**. 1/6 + 1/6 = 2/6 or 0.3333.
3. What is the probability of getting an odd number? Since half of our numbers are ODD, we have 3/6, or 1/2 = 0.5. Since 1, 3, and 5 are ODD, the OR rule still applies: 1/6 + 1/6 + 1/6 = 0.5.
4. What is the probability of getting a 3 AND a 5 on two rolls of the die? **When we specify AND, we must MULTIPLY the individual probabilities together**. 1/6 X 1/6 = 1/36 = 0.0278. Notice that OR situations have increased probability, but AND situations have decreased probability.

G. PERMUTATIONS and COMBINATIONS. The examples above were simple probability exercises. Often, probability is NOT so simple. We cannot, for example, use what we have learned to calcultate the odds of winning the lottery. Give the class the handout entitled COUNTING RULES. The top of the handout page identifies what we mean by FACTORIALS: **n factorial (denoted by n!) is the number of sequences in which n distinct objects can be arranged.** The way to calculate 5! would be to multiply 5 by 4 by 3 by 2 by 1 to get 120. The simplest way to do this is to use the **n!** key on the calculator. Show the class how this is done. The handout demonstrates that there are 4! = 24 ways to arrange the letters A, B, C, and D. This rule applies to the arrangement of ANY data set as well. How does this apply to calculting the odds of winning the lottery? The second page of the handout describes **PERMUTATIONS and COMBINATIONS. Permutations define the number of seqences possible when drawing x objects from a set of n distinct objects, when order IS important.** Discuss the formula and the example on the handout before proceeding. One common lottery game requires picking 3 numbers from a set of 10 numbers when order IS important. This simply means that if you chose 2,3,4 and the drawing resulted in 4,3,2 - you lose! The first question we must ask in calculating the odds would be, "How many three-number combinations are possible if order IS important?" Plugging into our equation results in 10! / 7! = 720. There are 720 possible combinations! If you buy ONE lottery ticket, your odds of winning are 1 in 720. There is another game of pick 3 in which order is NOT important. We use a COMBINATION to determine the odds. Refer back to the handout and review the definition and example. For this game, since order is NOT important, there are only 10! / (3! 7!) = 120 possible combinations. If you buy ONE ticket, the odds of winning are 1 in 120. What about the BIG lottery? You have to pick 6 numbers from a set of 50 numbers, but order is NOT important, so we have 50! / (6! 44!) = 15,890,700 possible combinations! Who wants to buy a ticket?!?

IV. PROBABILITY DISTRIBUTIONS. It has been established that probability is related to the frequency of an event occuring. **We can make a graphical representation of the frequency of an event in relation to the frequencies of all other possible outcomes, and this would be called a PROBABILITY DISTRIBUTION.**

A. Most industrial processes tend to follow what we call a **NORMAL** distribution. The distinguishing feature of this distribution is its **BELL SHAPED CURVE**. Bell shaped refers to the **PATTERN** of the distribution when graphed. Looking at a graph of the distribution, we can determine whether we PROBABLY have a problem with the data, or not, based on the uniformity of the PATTERN. Give the class the handout entitled PROBABILITY DISTRIBUTIONS. Note that the characteristics of the symetric distribution are such that the mean, median, and mode all have the same value. Based on the examples, we can see what happens to the pattern when these values are NOT the same. Going back to our original data, ask the class if they think our probability distribution would be symetric. **Since the mode (24.7) is less than the median (24.8), which is less than the mean (25.0), we could conclude WITHOUT EVEN PLOTTING THE DATA, that our distribution is skewed to the right.** That leaves us with at least **SOME** probability that a problem still exists within our process. A critical point to make here is that a probability distribution can ONLY suggest the probability of a problem occuring or not. **We cannot KNOW if there is a problem unless we physically examine our process.**

B. As stated, one of the distinguishing features of a normal distribution is the bell shaped curve. However, the curve itself is NOT our main interest. **It is the AREA under the curve that corresponds to probability.** Note that in a truly symetric distribution, exactly HALF the area under the curve lies below the mean, and half above. This corresponds to a 50% probability of a data point occuring in either of these areas.

C. The shape of the distribution is determined by the variation in the distribution. **If there was no variation, instead of a bell shaped curve, we would have a single data point.** Regardless of whether the distribution is symetric or skewed, the variation will also determine whether it is tall and narrow, or short and wide. **The most convenient statistical measure we have of the variation is the STANDARD DEVIATION, which refers to the average deviation of the data about the mean.** While there are complicated formulas involved with calculating the standard deviation of a set of data, a statistical calculator can be used to do this with ease.

1. Demonstrate to the class how to "get into" the statistical mode on the calculator. With most of these calculators, one must enter the data using the "data" key. Each time a data point is entered, the display will show the number of the data point - NOT THE VALUE. If there are 30 data points, the display should read n = 30 when the last data point has been entered. Have the class enter all of the data points from the original scenario.
2. The universal symbol for the **sample mean** is identified as $\overline{X}$ (X-bar). By pressing the $\overline{X}$ key on the key pad, the sample mean will be displayed. It should be the same as the class has already calculated, 25.0.
3. As the sample mean is an ESTIMATE of the population mean, the **sample standard deviation** is an estimate of the **population standard deviation**. The symbol for **population**

standard deviation is the lower case Greek letter sigma (σ) . The symbol for the **sample standard deviation** is simply an **s**. Pressing the **s** key on the keypad should give the sample standard deviation of 2.0055, which we can conveniently round to 2.0.

D. Another feature of the normal distribution is that it has points of inflection at the mean plus or minus multiples of the standard deviation. Give the class the handout entitled AREAS UNDER THE NORMAL CURVE. As shown, 99.73% of the area under the curve falls between the mean plus and minus three standard deviations. 95.44% of the area is between plus and minus two standard deviations, and 68.26% is between plus and minus one standard deviation. **THIS PERCENTAGE CORRESPONDS TO PROBABILITY!**

V. CONTROL CHARTS. **If our process is running on target with NO ASSIGNABLE CAUSES of variation, we can say that we are IN CONTROL of our process.**

A. If the process is IN CONTROL, it will follow a normal probability distribution, and the data collected from the process will be normal data. IF an assignable cause of variation happens in our process, the overall effect will be to shift the mean, and thus skew the distribution. When this happens, we say that we are OUT OF CONTROL. When we have evidence to suggest that an assignable cause of variation has occured, we should investigate our process to identify and correct this cause, thus bringing our process back into control. The reason the word CONTROL is used is very simple. When we **Monitor** our processes, **Detect** a possible problem, **Investigate** the process, and **Correct** the problem, we are CONTROLLING the process to the desired target. We are following what is referred to as the **MDIC** loop.

B. If we plot the data we have collected in the form of a probability distribution, we would be very disappointed because we simply do not have enough data points to adequately approach a normal bell shaped curve. We would have to have at least 100 data points to get a good picture, and even then the curve may suggest problems with the data that do not exist. However, we can do what most industrial facilities do - we can use our data to develop a CONTROL CHART. **A control chart is actually an extrapolation of the bell shaped curve.** Just as the bell shaped curve has points of inflection at the mean plus or minus multiples of the standard deviation, the control chart has the same points of inflection. The probabilities associated with these points of inflection also apply to the control chart.

C. Give the class the handout entitled **CONTROL CHARTS**. This handout demonstrates how a bell shaped curve is actually transformed into a control chart. The points of inflection serve as CONTROL LIMITS. The limits at the mean plus and minus two standard deviations are called 2-sigma (2σ) limits. The outer limits are called 3-sigma (3σ) limits. Have the class label the mean they calculated from the data. Since they have already calculated the standard deviation to be 2.0, they can identify the 2- and 3-sigma limits on their control charts. Have them label the rest of the axis and plot the data. In a normal distribution, 95.44% of the data will fall between the 2-sigma limits. This means that IF there is a data point outside these limits, we have a 95.44% probability that an assignable cause of variation has occured. Similarly, if a data point falls outside the 3-sigma limits, we have a 99.73% probability that an assignable cause of variation has occured. Some industrial processes are best controlled using 2-sigma limits, while others use 3-sigma limits. When 3-sigma limits are used, the 2-sigma limits are often referred to as WARNING LIMITS. Once a determination has been made as to whether to use 2-sigma control limits, or 3-sigma, a point outside the control limits would indicate that the process is **PROBABLY** out of control. If all the points are inside the limits, the process is **PROBABLY** in control. Reinforce the concept that **a control chart WILL NOT tell us if there is or is not a problem with the process, it will ONLY tell us that there PROBABLY is or is not.** For this reason, we choose NOT to make changes to a process until the probability of a problem is very high, 95% or more. If we change the process when it is ACTUALLY in control, that action becomes an assignable cause of variation and WILL cause the process to shift out of control. Even if the probability is very high, we should always investigate the process before actually changing anything.

D. Our control limits were calculated to help us determine if there was still a problem with our process. Therefore, we are best concerned with 3-sigma limits, as is most often the case. We see that none of the data points are outside the control limits, and only one data point is below the LOWER warning limit. We can conclude that our process is probably in control and leave it alone. We could further conclude that at the 95% level of significance, there is room for future improvements to our process.

E. Give the class the **CONTROL CHARTS EXERCISE**. When a point is OUT OF CONTROL, the first thing we do is **CIRCLE IT**. That helps to call attention to it on the control chart. Ask the class to identify and circle all out of control points on the hand-out. Review these with the class by referring to the date of the out of control data points.

F. Points outside the limits are not the only means of using a control chart to determine if our process is probably out of control. Referring back to the points of inflection on the bell shaped curve, we see that 68% of our data should fall in the region identified between the mean plus and minus one standard deviation. There is no such thing as a control chart with 1-sigma limits. BUT, this being the case, **whenever we have 8 points in a row ABOVE or BELOW mean, evidence**

suggests that we have violated this space, and are probably out of control. It is actually an indication that our mean has shifted due to an assignable cause of variation. **We should treat the 8th point the same way we would treat a point outside the limits**. Have the class identify (and circle) any out of control points that meet this description on their exercise.

G. Circling the out of control point is obviously not going to get the process back in control. When we have identified the data point, we must then investigate the process to see if we can identify the cause. Since ten people operating a process may have ten different ways of investigating, we need to have a standard way of conducting the investigation. We also need this standard to lead us to the problem as accurately as possible in the shortest amount of time. Industrial facilities do this by bringing together everyone involved with the process and using their expertise to develop the best possible strategy to follow in the investigation of the process. This becomes the **CONTROL STRATEGY**, and is kept with the control chart. **It tells us what to check, when to check it, and often, how to check it**. Control strategies are tested whenever we have to use them to investigate our process. Did the strategy lead us to the problem? Was the path to the problem straigtforward, or did we waste time getting to the correct problem? Keeping track of problems and solutions in a log will build a data base for us to use in reviewing and updating our control strategies.

H. **When we follow the MDIC loop (Monitor, Detect, Investigate, and Correct), we are CONTROLLING OUR PROCESS**. The tool we use for the first half of the loop, Monitor and Detect, is the **CONTROL CHART**. The tool we use for Investigate and Correct, is the **CONTROL STRATEGY**. We collected a lot of data and determined the limits of a control chart with a statistical calculator. Most control charts are developed the same way. The important part to see is that even though we need 30 or 40 data points to determine the limits, we can then analyze our process every time we have a subsequent data point to plot. We do not have to collect our data in large numbers once the limits have been calculated. **Each data point gives us an instant "snapshot" of the process.** The data recorded may be instrument readings related to the process or lab results obtained by sampling the process. In many labs, control charts are developed for analytical procedures by running lab standards or reference materials. Once the control chart is in place, the standards are run daily or every shift, and the control chart is used to determine whether or not anything has changed.

I. **Regardless of the application, an important consideration to keep in mind is that the limits are derived from the process itself, not the customer specifications.** Customer specs tell us if the materials we produce are adequate for the customer's needs. Control limits tell us whether or not our process is in need of attention.

J. In closing, point out that when we compared the mean, median, and mode of our data, we saw there was at least **SOME** probability that a problem still exists in the process. **By calculating the standard deviation and developing a control chart, we saw that this probability is within acceptable limits.** At this point, the control chart COULD be placed into service, and the process monitored daily to see if it remains in control. One important consideration would be to keep in mind that **even when we are IN CONTROL, we need to keep a constant vigil for ways to improve the process**.

VI. As an exercise, divide the class into as many groups as possible to have one vernier caliper per group. Give each student a bag of M&M candy. Each student should use the vernier caliper to measure the **thickness** of five (5) of their M&M candies, and record the results. Point out that it is the **SMALLER** of the two cross sections of the candies we are trying to measure. It would probably be best if the instructor demonstrates how to do this before asking the class to do so. When the measurements have been made, and the data recorded, a histogram will be developed from the data. The easiest way to facillitate this would be to estimate the mean from two or three sample data sets and draw a vertical axis on the whiteboard (chalkboard) around the mean, with all possible measurements labeled on the axis. As each student reports (verbally) their personal results, the instructor (or any class volunteer) should mark an X beside the corresponding data point on the axis. At the same time, a volunteer should be recording EACH data point so that there will be a master list compiling ALL the data collected. The class need not have a working knowledge of what a histogram is, but the instructor should. When the histogram is complete, a curve can be drawn around the X's to show the probability distribution. The data collected can then be entered into the statistical calculator to determine the standard deviation, from which can be derived the control limits. As the exercise progresses, the instructor should reinforce concepts previously taught.

TABULATED PROCESS DATA FROM SCENARIO

The Following Data Was Collected by Measuring the Amount of Time Necessary to Fabricate One (1) Video Game Unit from Start to Finish (in minutes).

1. 23.4
2. 22.5
3. 25.0
4. 26.4
5. 24.7
6. 23.8
7. 26.0
8. 27.5
9. 26.8
10. 24.0
11. 23.5
12. 20.7
13. 25.0
14. 24.7
15. 21.4
16. 28.0
17. 26.4
18. 23.6
19. 25.4
20. 29.0
21. 24.2
22. 22.9
23. 25.7
24. 26.4
25. 24.7
26. 22.5
27. 25.7
28. 27.0
29. 28.4
30. 24.7

CAUSES OF VARIATION EXERCISE

In the spaces beside each possible cause of variation in **monthly electricity bills**, identify **ASSIGNABLE** causes with an **A**, and **EXPECTED** causes with an **E**.

* **REMEMBER:** Assignable causes can usually be identified as those REASONS for large differences, while Expected causes are usually RANDOM, and cause very small differences.

_____ A very HOT month

_____ Watching more or less television

_____ Using extra ice

_____ Air conditioner is broken

_____ Incorrect billing

_____ Using the electric oven more or less

_____ Taking a month long vacation - house empty

_____ Having children visit for a couple of days who run in and out of doors

_____ Rate increase

_____ Leaving coffee maker on overnight

_____ Having a house full of company for an extended period of time

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__A___ A very HOT month

__E___ Watching more or less television

__E___ Using extra ice

__A___ Air conditioner is broken

__A___ Incorrect billing

__E___ Using the electric oven more or less

__A___ Taking a month long vacation - house empty

__E___ Having children visit for a couple of days who run in and out of doors

__A___ Rate increase

__E___ Leaving coffee maker on overnight

__A___ Having a house full of company for an extended period of time

ORDERED DATA FROM SCENARIO

1. 20.7
2. 21.4
3. 22.5
4. 22.5
5. 22.9
6. 23.4
7. 23.5
8. 23.6
9. 23.8
10. 24.0
11. 24.2
12. 24.7
13. 24.7
14. 24.7
15. 24.7
16. 25.0
17. 25.0
18. 25.4
19. 25.7
20. 25.7
21. 26.0
22. 26.4
23. 26.4
24. 26.4
25. 26.8
26. 27.0
27. 27.5
28. 28.0
29. 28.4
30. 29.0

COUNTING RULES

FACTORIALS

n factorial - denoted by **n!** - is the number of sequences in which n distinct objects can be arranged

$$n! = n(n-1)(n-2)...(3)(2)(1)$$

for any positive integer n

EXAMPLE

There are 4! = 4(3)(2)(1) = 24 ways to arrange the four letters

A,B,C, and **D**

ABCD	**ABDC**	**ADCB**	**ACBD**	**ACDB**	**ADBC**
BACD	**BADC**	**BCAD**	**BCDA**	**BDAC**	**BDCA**
CABD	**CADB**	**CBCA**	**CBAD**	**CDAB**	**CDBA**
DABC	**DABC**	**DBAC**	**DBCA**	**DCAB**	**DCBA**

COUNTING RULES

PERMUTATIONS

Permutations define the number of sequences possible when drawing **x** objects from a set of **n** distinct objects when

order IS important $$P_x^n = \frac{n!}{(n-x)!}$$

EXAMPLE

There are $P_2^4 = \frac{4!}{(4-2)!} = \frac{4!}{2!} = \frac{24}{2} = 12$ ways of choosing 2-letter sequences from a set of 4 letters (**A,B,C,D**):

AB AC AD BC BD CD

BA CA DA CB DB DC

COMBINATIONS

Combinations define the number of ways to draw **x** objects from a set of **n** distinct objects when **order is NOT important**

$$C_x^n = \binom{n}{x} = \frac{n!}{x!(n-x)!}$$

EXAMPLE

There are $C_2^4 = \binom{4}{2} = \frac{4!}{2!(4-2)!} = 6$ possible 2-letter combinations:

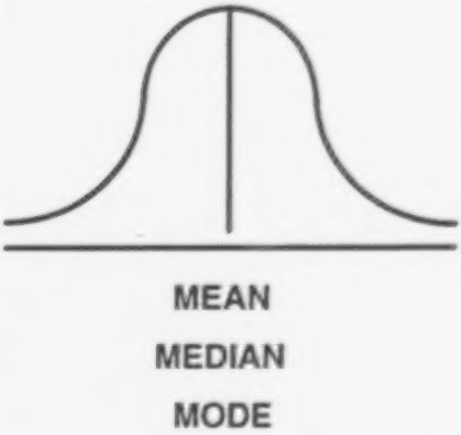
AB AC AD BC BD CD

PROBABILITY DISTRIBUTIONS

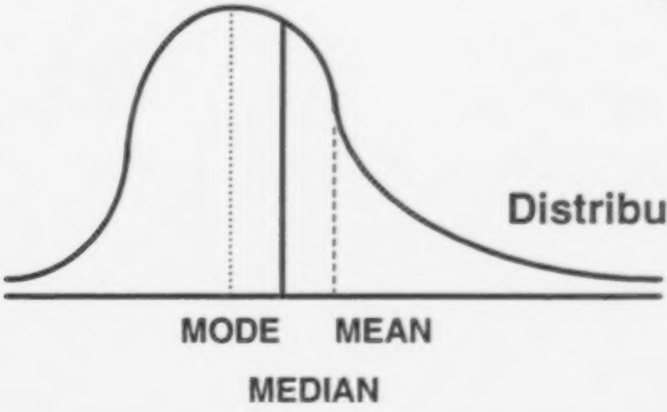
Symmetric Distribution

mean, median, and mode

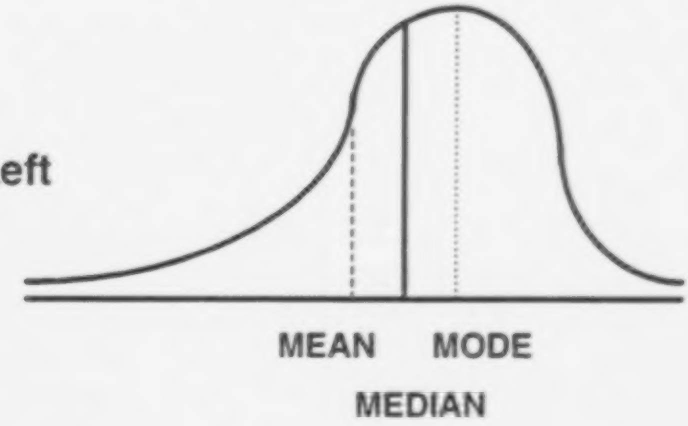
all at same point


MEAN
MEDIAN
MODE

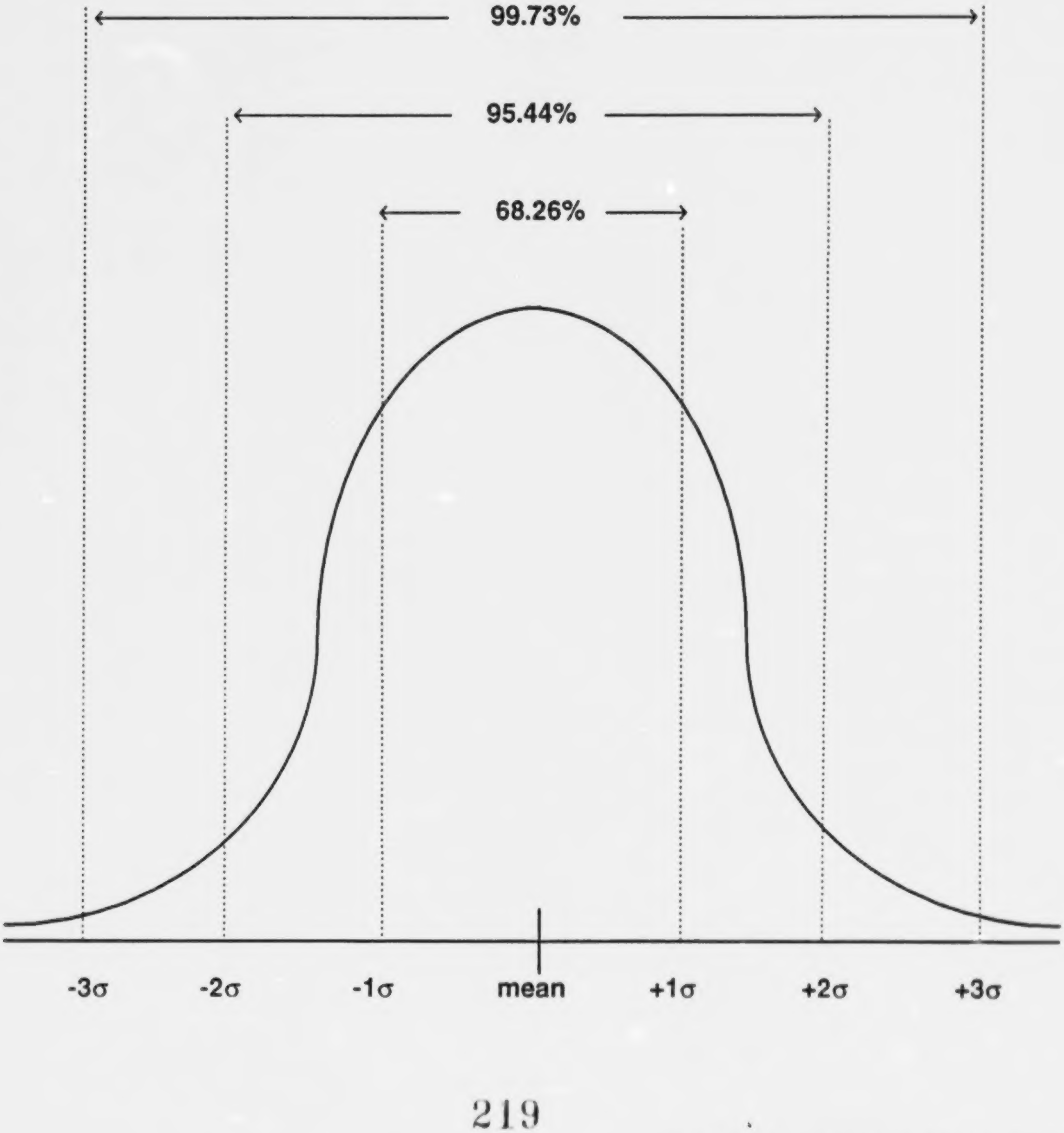
Distribution Skewed Right


MODE
MEAN
MEDIAN

Distribution Skewed Left

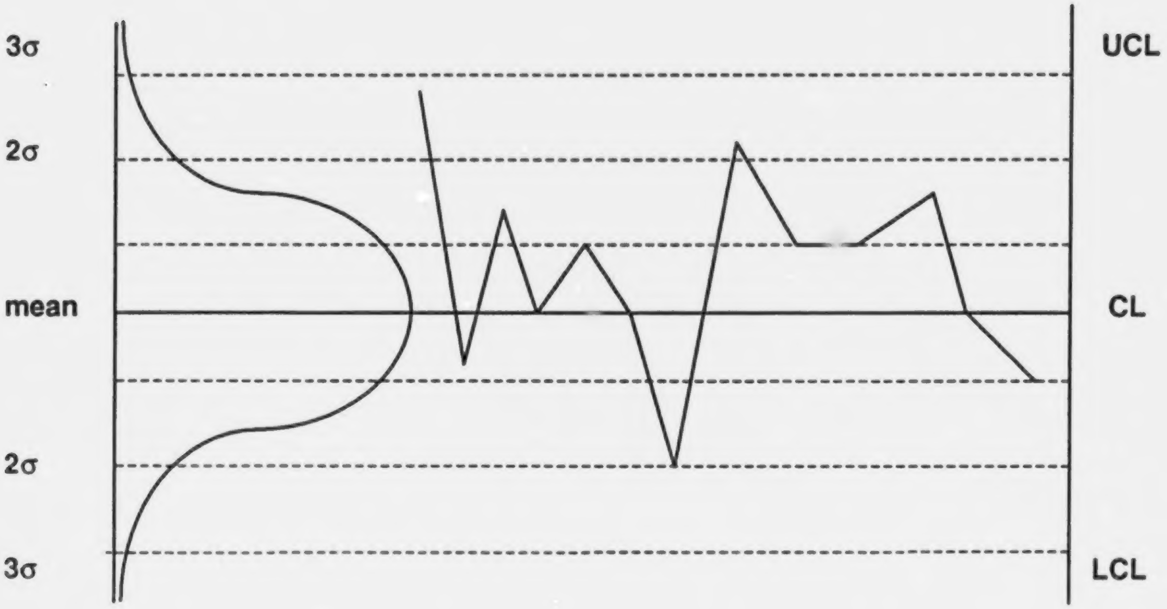

MEAN
MODE
MEDIAN

AREAS UNDER THE NORMAL CURVE

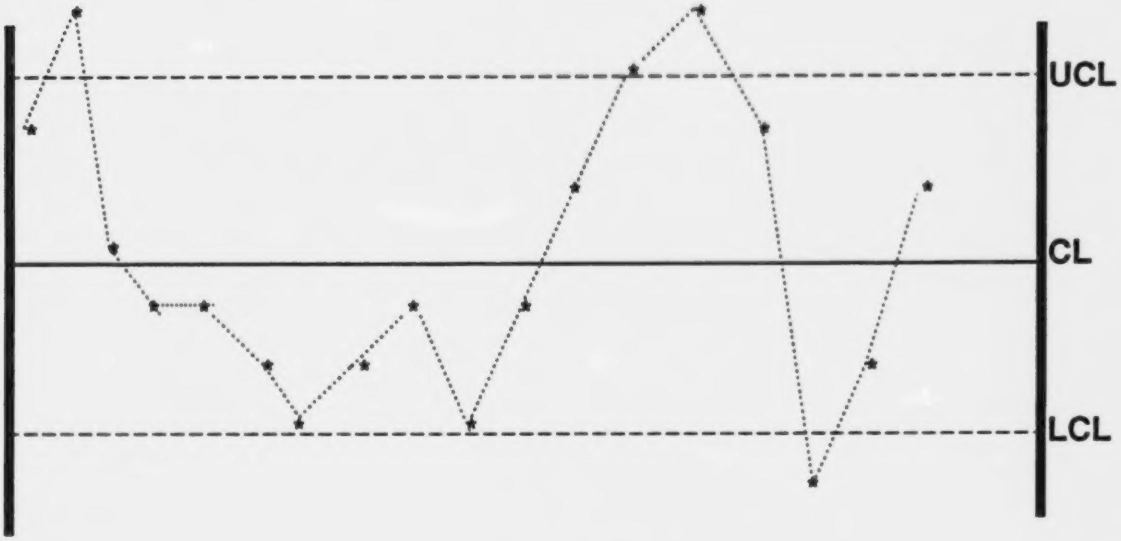

99.73%
95.44%
68.26%
-3σ
-2σ
-1σ
mean
+1σ
+2σ
+3σ

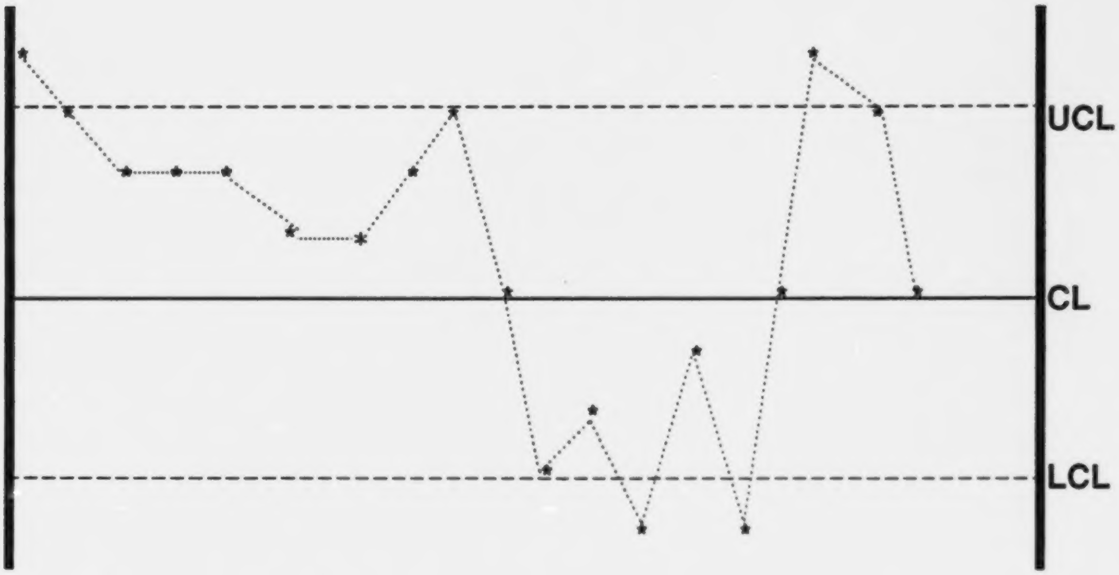
219

CONTROL CHARTS


3σ
UCL
2σ
mean
CL
2σ
3σ
LCL

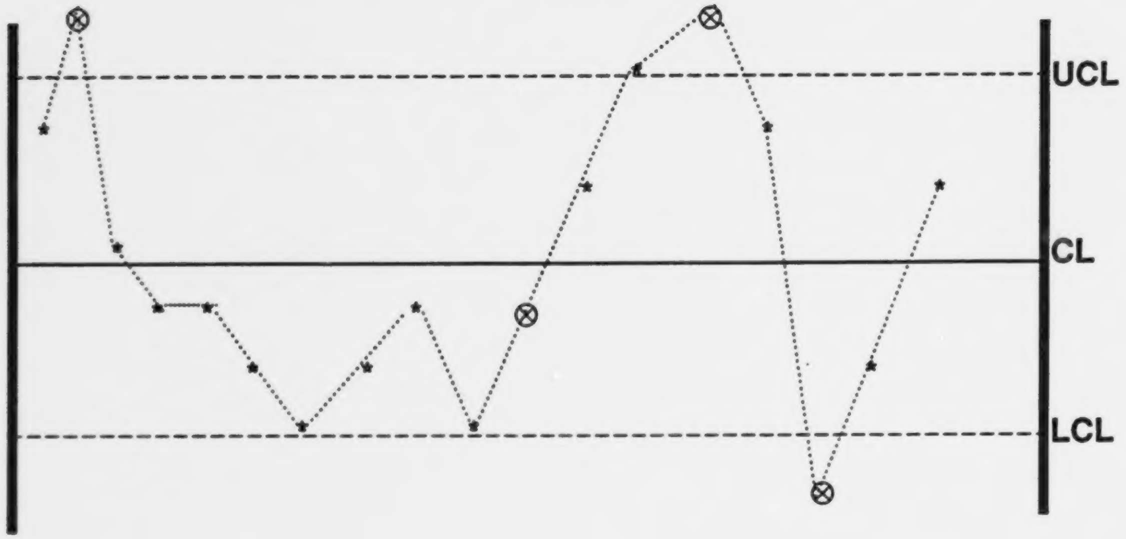
CONTROL CHARTS EXERCISE

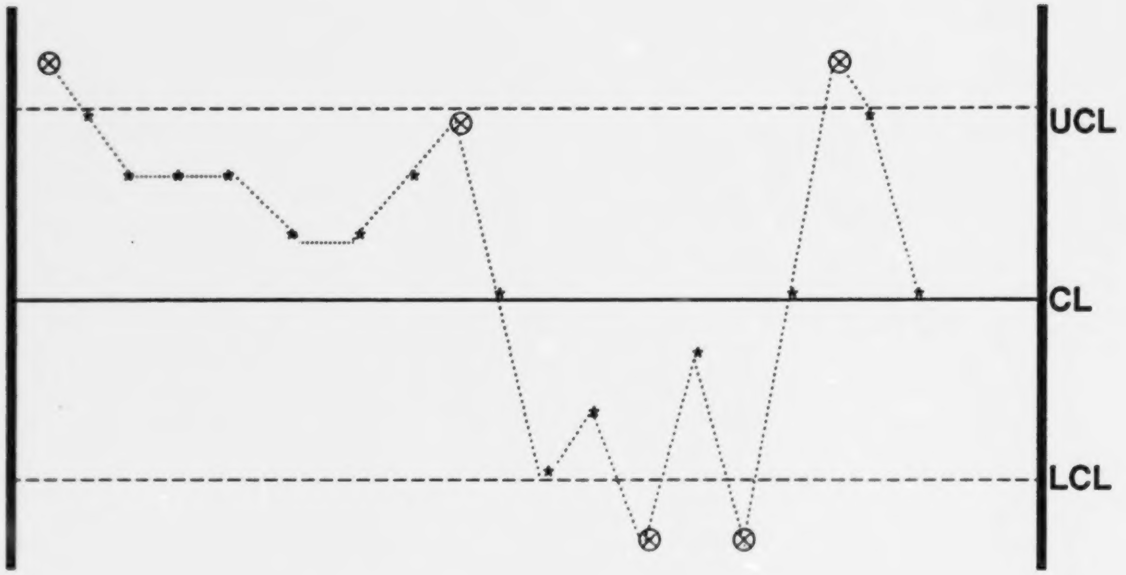




CONTROL CHARTS EXERCISE

(ANSWER KEY)





222

COURSE OBJECTIVES

1. The student will be able to define (in writing) the following terms: SPC, PROCESS, SAMPLING, and VARIATION.
2. Given a list of possible causes of variation between data, the student will be able to identify which causes are ASSIGNABLE, and which are EXPECTED.
3. The student will be able to define the terms PRECISION and ACCURACY, and will be able to explain how these terms are related to VARIATION.
4. The student will be able to idedntify a normal distribution by the shape of the distribution curve.
5. Given a calculator and a set of normal data, the student will be able to calculate the MEAN, MEDIAN, and MODE of the data set.
6. The student will be able to identify the components of a control chart. (Mean charts only - range charts will not be covered in this lesson plan.)
7. Given a control chart with data points plotted, the student will be able to identify (circle) out-of-control points.
8. Given a statistical calculator and a set of normal data, the student will be able to calculate the MEAN and STANDARD DEVIATION of the data, and use these statistics to determine 2- and 3-sigma control limits.
9. The student will be able to explain the purpose of control charts and control strategies, and how they are used to control processes.

EVALUATION

(SKILL CHECK)

1. Correctly define each of the following terms:

   a. SPC ____

   ____

   b. PROCESS ____

   ____

   ____

   c. SAMPLING ____

   ____

   ____

   d. VARIATION ____

   ____

   ____

   e. PRECISION ____

   ____

   ____

   f. ACCURACY ____

   ____

   ____

2. The liquid level in a production tank is changing. Identify each of the following possible causes of variation with an **A** for **ASSIGNABLE**, and an **E** for **EXPECTED**.

   _____ Material is being added to the tank

   _____ Minor fluctuations in level controller

   _____ Tank is leaking

   _____ Differences in surrounding air temperature

   _____ Incorrect level measurements

224

SKILL CHECK - continued

3. Use a statistical calculator to answer the following questions for the data set below.

a. What is the **MEAN**? ______________________

b. What is the **MEDIAN**? ______________________

c. What is the **MODE**? ______________________

e. What is the **STANDARD DEVIATION**? ______________________

f. What are the **3-SIGMA LIMITS?** **UCL**__________ **LCL**__________

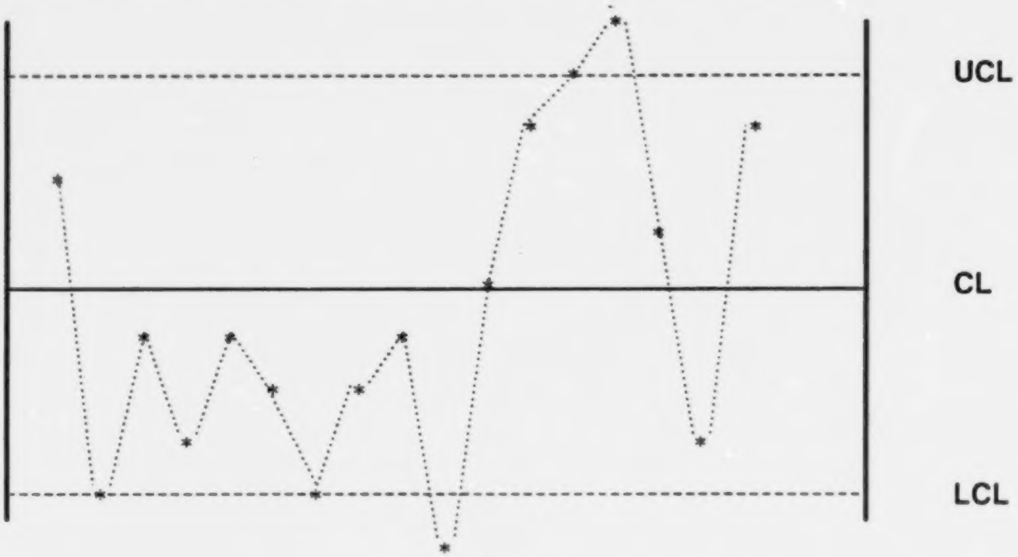
g. What are the **2-SIGMA LIMITS?** **UCL**__________ **LCL**__________

DATA SET:

38.5	**36.5**	**37.5**	**39.0**	**37.5**	**36.0**	**39.5**
37.0	**39.0**	**38.0**	**37.5**	**35.5**	**37.0**	**37.5**
36.5	**38.0**	**37.5**	**40.0**	**37.0**	**36.0**	**39.0**

4. A control chart based on **3-sigma limits** has a **mean** of 430.0 psi, and a **UCL** of 450.4 psi, and an **LCL** of 409.6 psi. What is the **STANDARD DEVIATION** of the process? ______________________

5. **CIRCLE** all **out-of-control** points on the following control chart.



225

EVALUATION

(SKILL CHECK)

1. Correctly define each of the following terms:

a. SPC STATISTICAL PROCESS CONTROL

b. PROCESS any and all steps necessary to achieve some end.

c. SAMPLING looking at part of something in order to make a decision about all of it.

d. VARIATION differences among items from the same process

e. PRECISION the degree to which repeated measurements of the same unit result in the same value

f. ACCURACY the closeness of agreement between the observed value and the correct value

2. The liquid level in a production tank is changing. Identify each of the following possible causes of variation with an **A** for **ASSIGNABLE**, and an **E** for **EXPECTED**.

A Material is being added to the tank

E Minor fluctuations in level controller

A Tank is leaking

E Differences in surrounding air temperature

A Incorrect level measurements

226

SKILL CHECK - continued

3. Use a statistical calculator to answer the following questions for the data set below.

   a. What is the **MEAN**? 37.6

   b. What is the **MEDIAN**? 37.5

   c. What is the **MODE**? 37.5

   e. What is the **STANDARD DEVIATION**? 1.2

   f. What are the **3-SIGMA LIMITS?** UCL 41.2 LCL 34.0

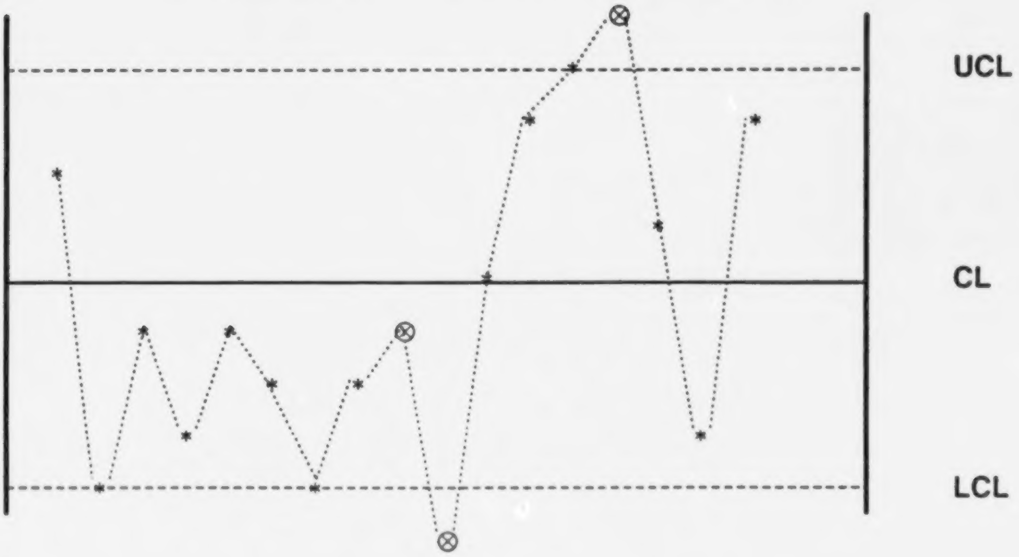
   g. What are the **2-SIGMA LIMITS**? UCL 30.0 LCL 25.2

   DATA SET:

38.5	**36.5**	**37.5**	**39.0**	**37.5**	**36.0**	**39.5**
37.0	**39.0**	**38.0**	**37.5**	**35.5**	**37.0**	**37.5**
36.5	**38.0**	**37.5**	**40.0**	**37.0**	**36.0**	**39.0**

4. A control chart based on **3-sigma limits** has a **mean** of 430.0 psi, and a **UCL** of 450.4 psi, and an **LCL** of 409.6 psi. What is the **STANDARD DEVIATION** of the process? (450.4 - 430.6) ÷ 3 = 6.8

5. **CIRCLE** all **out-of-control** points on the following control chart.



GLOSSARY

ACCURACY - the closeness of agreement between an observed value and the correct value.

COMBINATION - the number of sequences possible when drawing x objects from a set of n distinct objects when order IS NOT important.

CONTROL CHART - a graph of the data centered around the mean with points of inflection at the mean plus or minus multiples of the standard deviation, used to control processes.

CONTROL STRATEGY - a strategy or plan developed to investigate processes whenever a control chart has indicated that the process is probably out of control.

HISTOGRAM - a graphical representation of a frequency (probability) distribution.

IN CONTROL - on target, with no assignable variation.

MDIC LOOP - Monitor, Detect, Investigate, Correct - what we are doing when we use control charts and control strategies to control our processes.

MEAN - a statistical measure of the average of the data.

MEDIAN - the "middle" observation of the ordered data.

MODE - the value which occurs most often in the data set.

PERMUTATION - the number of sequences possible when drawing x objects from a set of n distinct objects when order IS important.

POPULATION - the total collection of units from a common source.

PRECISION - the degree to which repeated measurements of the same unit result in the same value.

PROBABILITY - the ratio of the number of times an event occurs to the number of chances it has to occur.

PROBABILITY DISTRIBUTION - a distribution of the data that represents the frequency of an event in relation to the frequencies of all other possible outcomes.

PROCESS - any and all steps or activities necessary to achieve some end.

SAMPLE - a subset of units collected from the population.

SAMPLING - looking at part of something in order to make a decision about all of it.

SPC - Statistical Process Control. A method of using statistics to control processes.

STANDARD DEVIATION - a statistical measure of the average deviation of the data about the mean.

VARIATION - the differences between items from the same process.